

TENSION MEMBER

BY

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SECTION 6 DESIGN OF TENSION MEMBERS

6.1 Tension Members

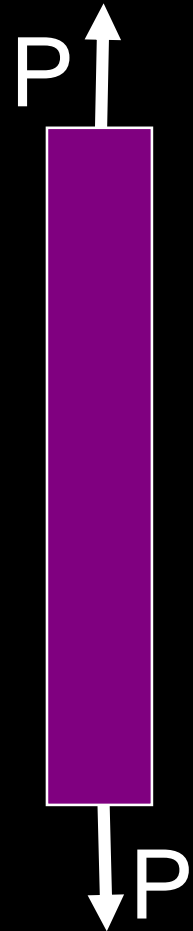
6.2 Design Strength due to Yielding of Gross Section

6.3 Design Strength due to Rupture of Critical Section

- 6.3.1 Plates
- 6.3.2 Threaded Rods
- 6.3.3 Single Angles
- 6.3.4 Other Sections

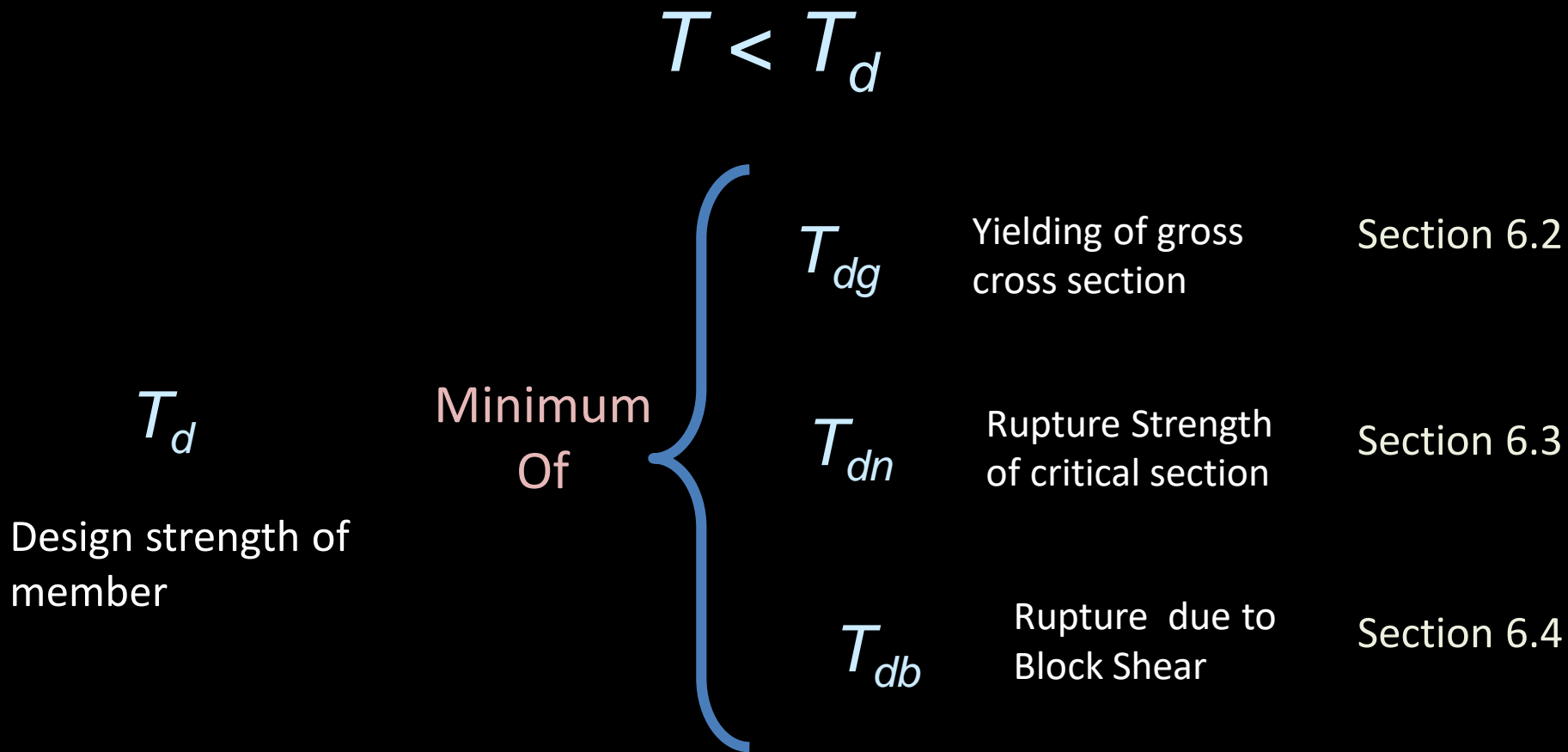
6.4 Design Strength due to Block Shear

- 6.4.1 Bolted Connection
- 6.4.2 Welded Connection



6.1 Tension Members

The factored design tension T , in the members



6.2 Design Strength due to Yielding of Gross Section

$$T_{dg} = f_y A_g / \gamma_{m0}$$

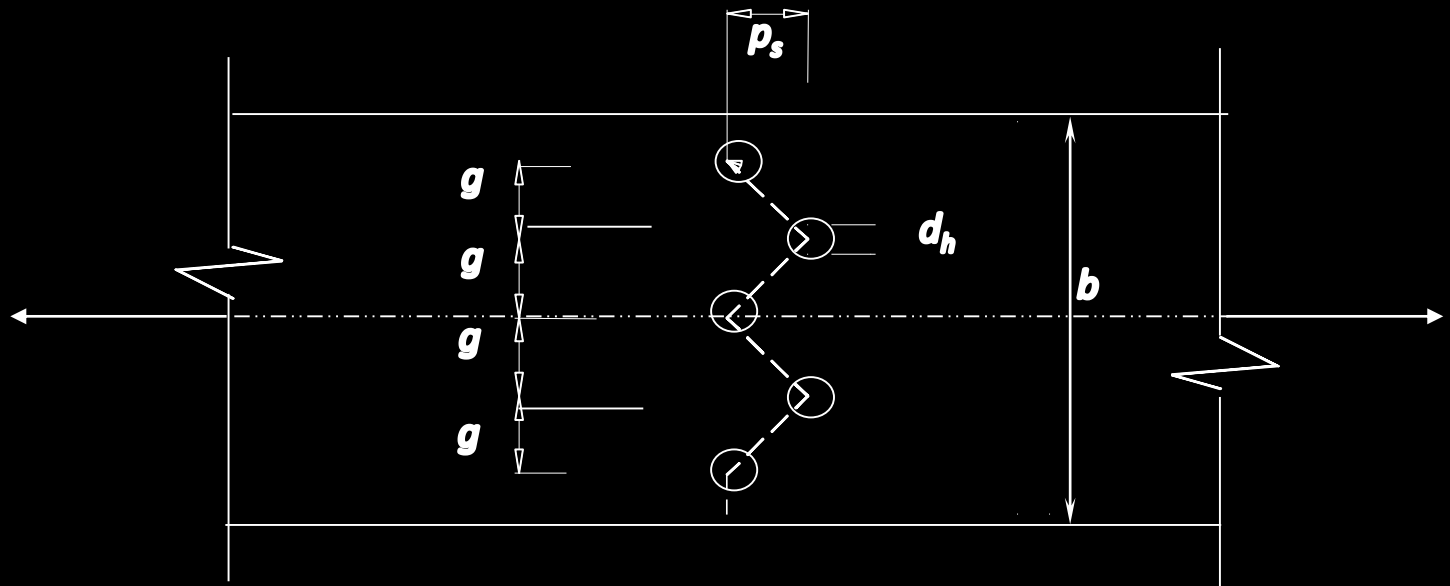
$$\gamma_{m0} = 1.1$$

6.3 Design Strength due to Rupture of Critical Section

6.3.1 Plates – The design strength in tension of a plate, T_{dn} ,

$$T_{dn} = 0.9 f_u A_n / \gamma_{m1} \quad \gamma_{m1} = 1.25$$

$$A_n = \left[b - nd_h + \sum_i \frac{p_i^2}{4g_i} \right] t$$

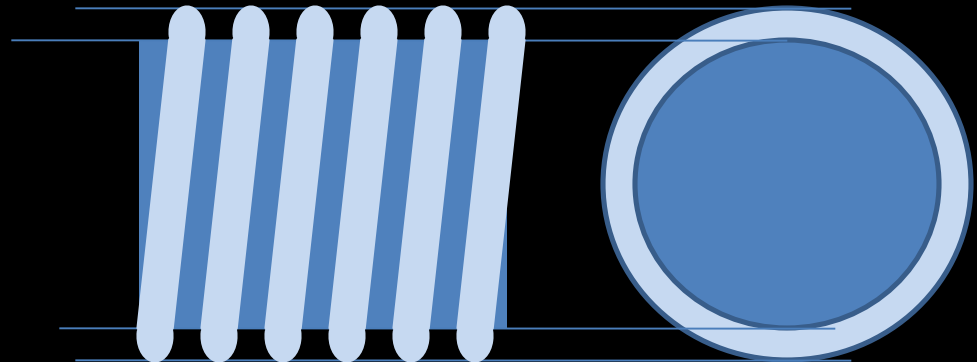


6.3 Design Strength due to Rupture of Critical Section

6.3.2 Threaded Rods –

The design strength of threaded rods in tension, T_{dn} ,

$$T_{dn} = 0.9 f_u A_n / \gamma_{m1}$$



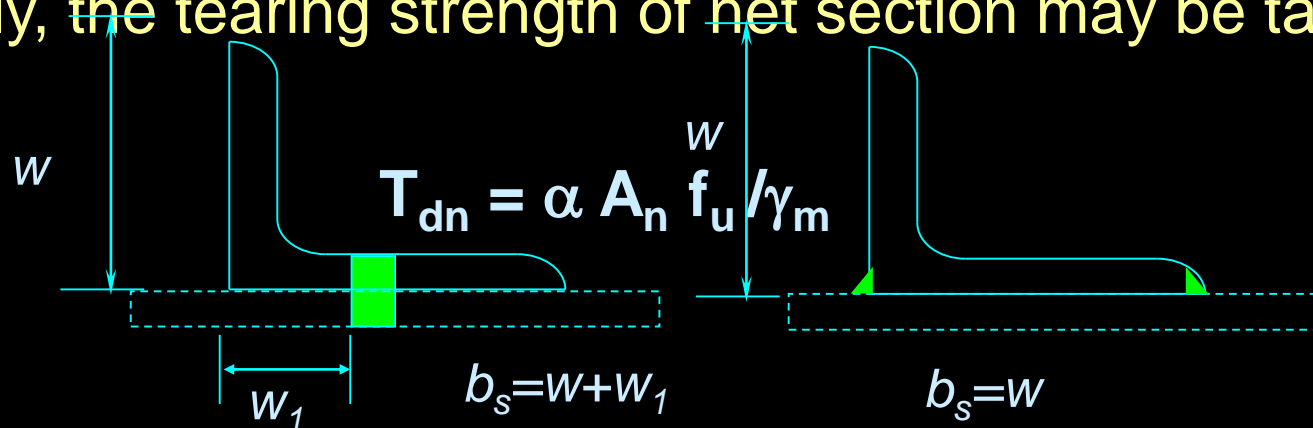
6.3 Design Strength due to Rupture of Critical Section

6.3.3 Single Angles – The design strength, T_{dn} , as governed by shear lag

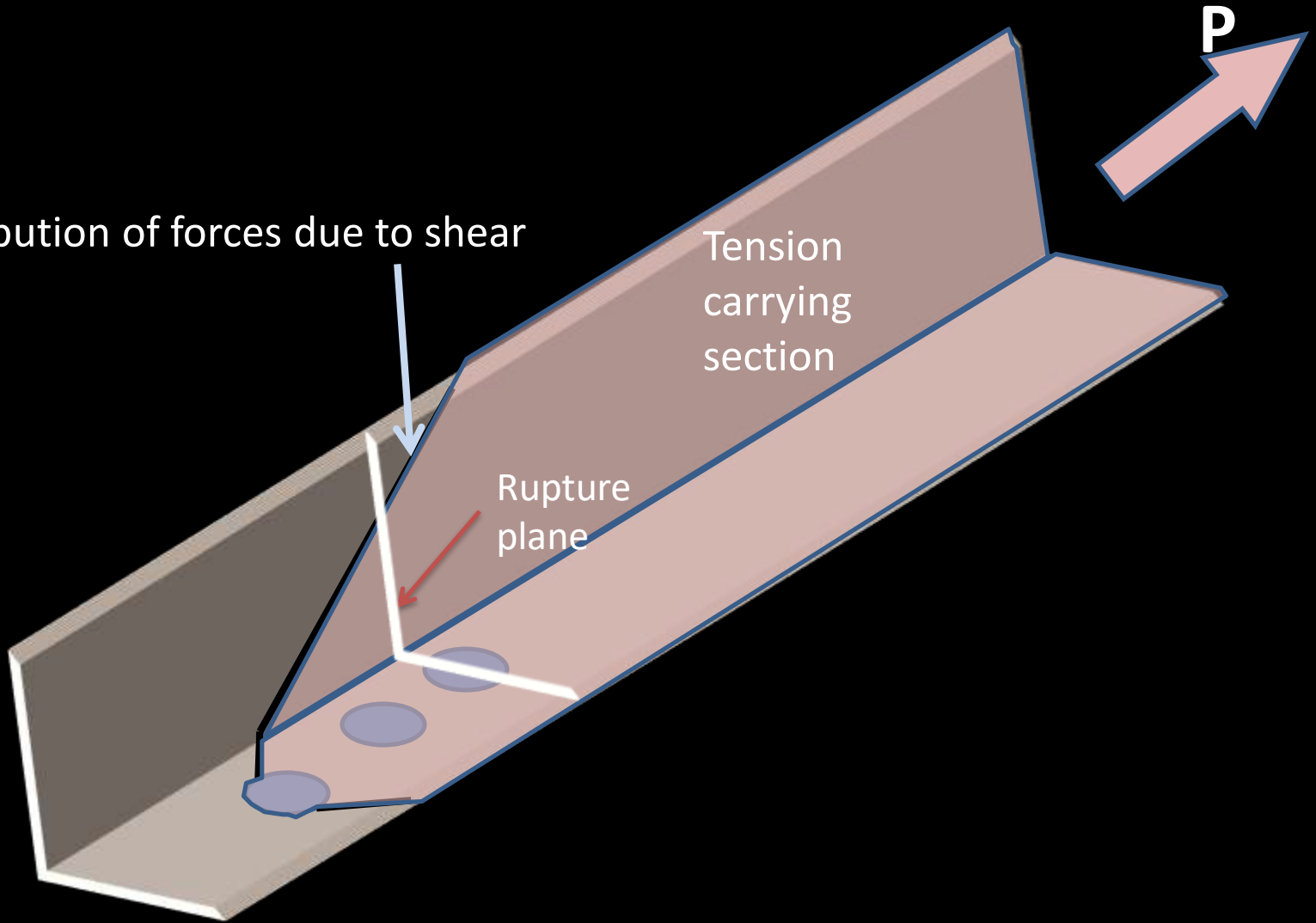
$$T_{dn} = 0.9 f_u A_{nc} / \gamma_{m1} + \beta A_{go} f_y / \gamma_{m0}$$

$$\beta = 1.4 - 0.076 (w/t) (f_u/f_y) (b_s/L) \quad [\approx 1.4 - 0.52(b_s/L)]$$

Alternatively, the tearing strength of net section may be taken as



Distribution of forces due to shear lag

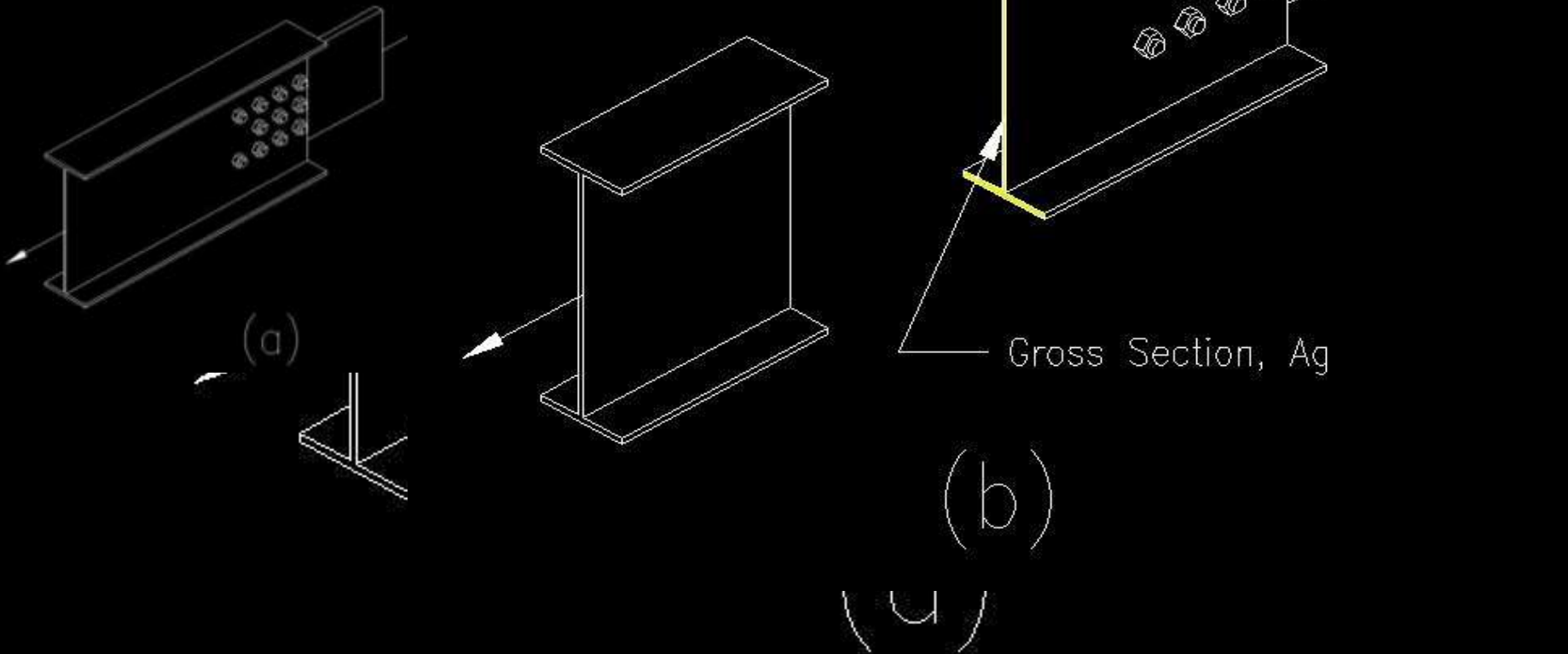


Tension carrying section

Rupture plane

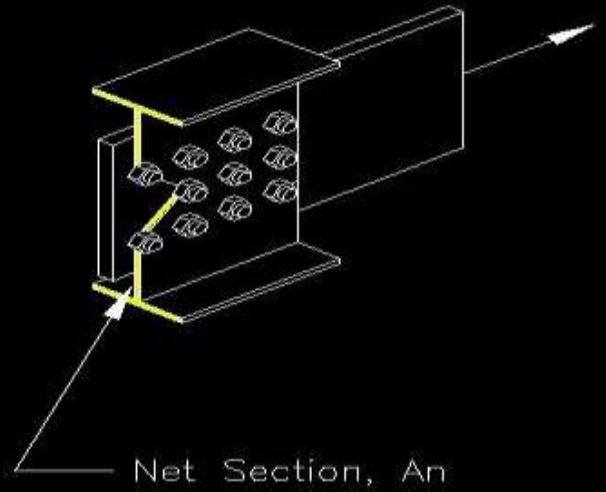
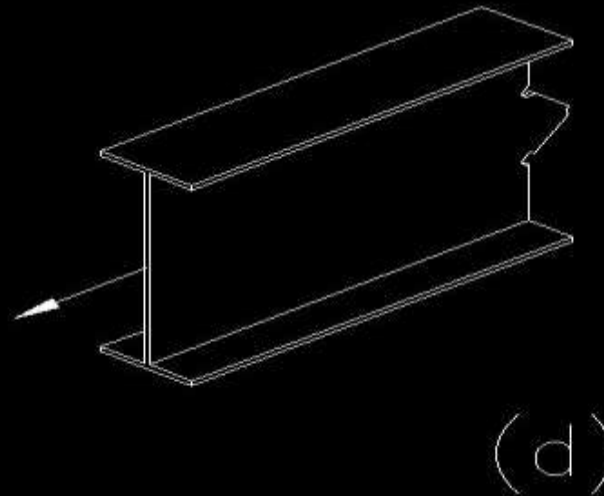
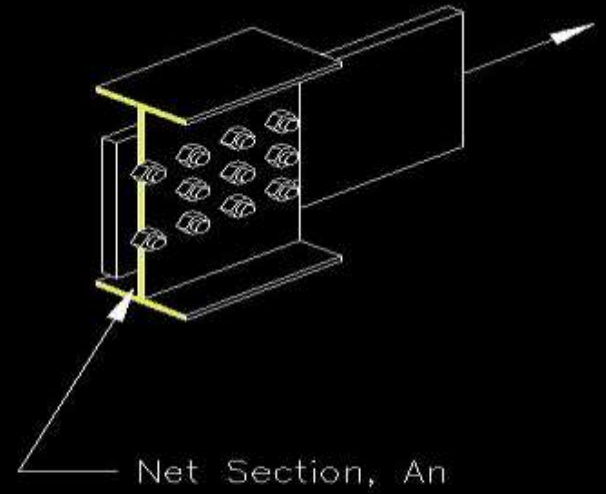
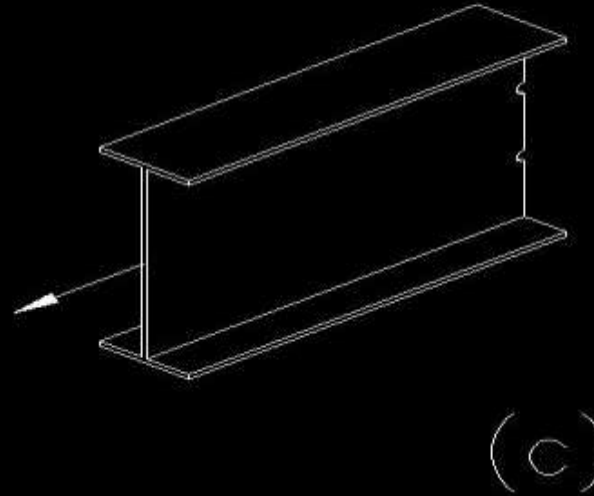
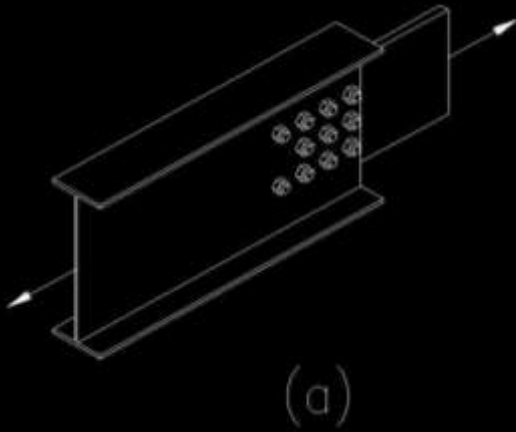
Example

Yielding due to
gross cross section



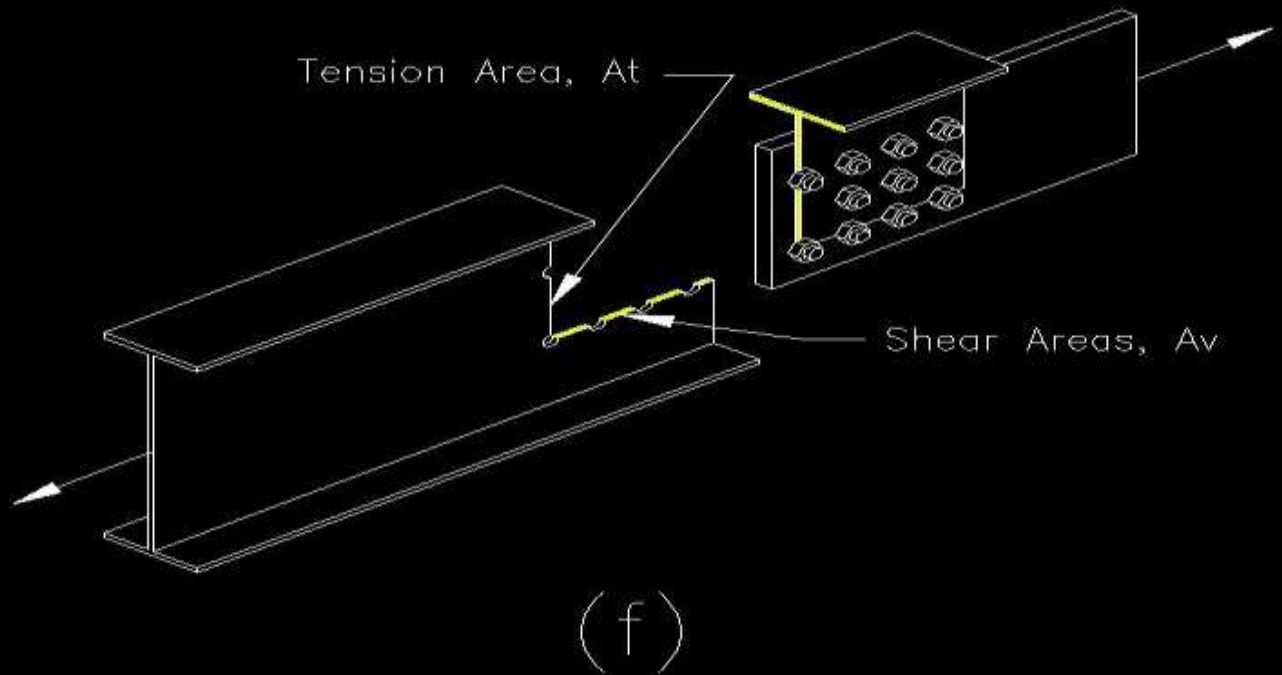
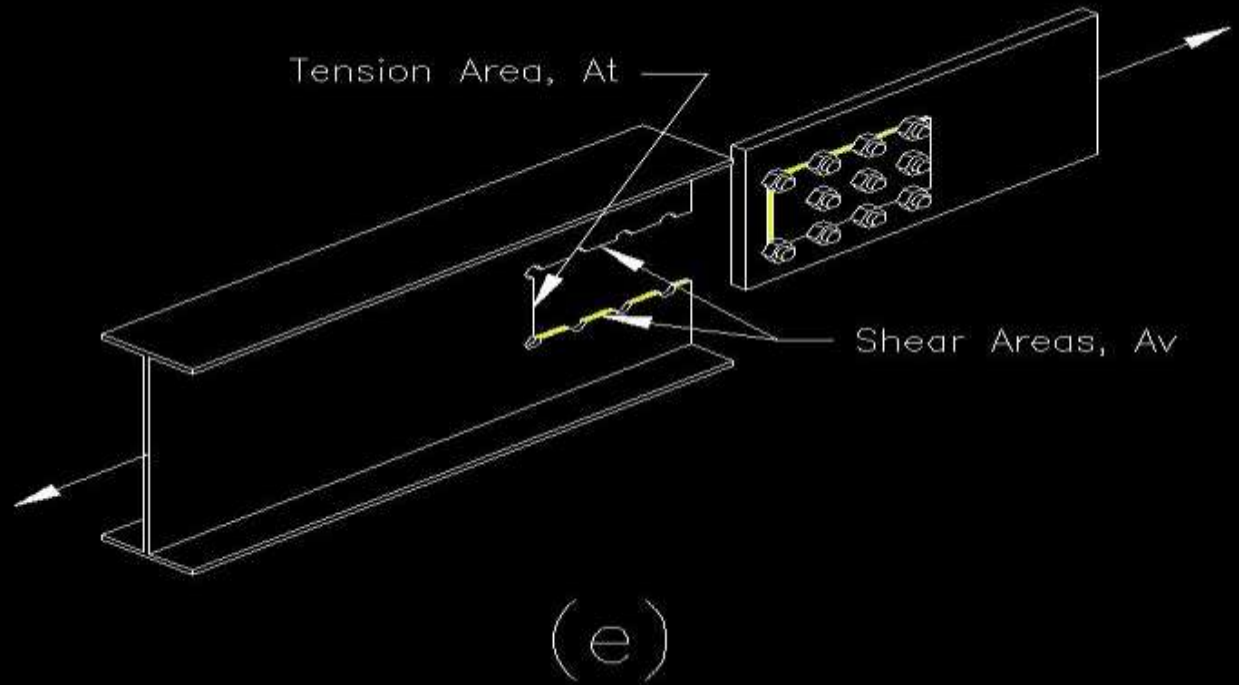
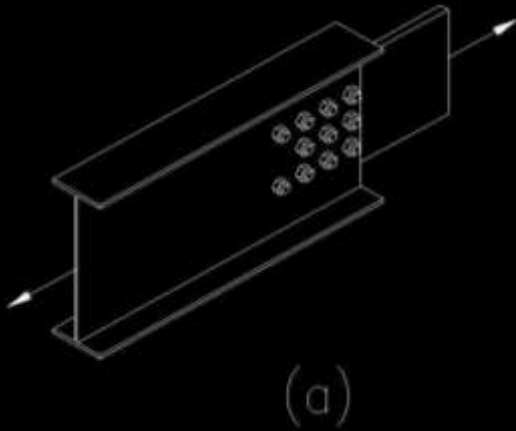
Example

Rupture Strength
of critical section



Example

Rupture due to
Block Shear

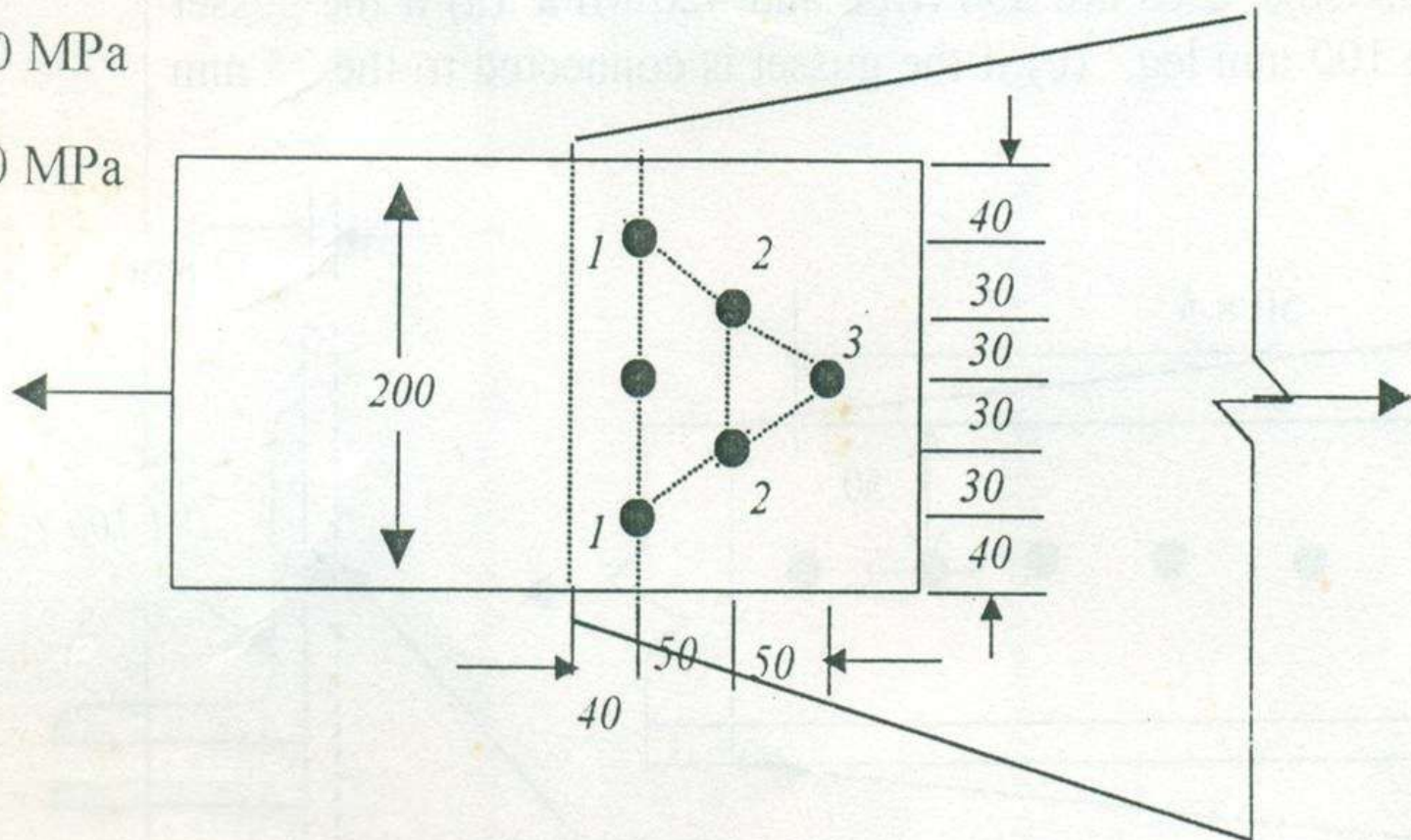


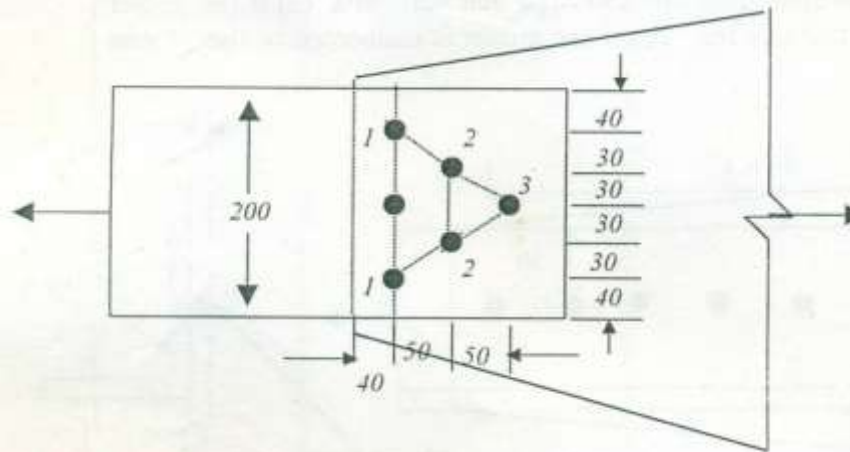
Tension member design: Example -1

Determine the design tensile strength of the plate (200 × 10 mm) connected to 12 mm thick gusset, using 20 mm bolts as shown below, if the yield and the ultimate stress of the steel used are 250 MPa and 420 MPa, respectively

$$f_y = 250 \text{ MPa}$$

$$f_u = 420 \text{ MPa}$$





$$A_n \text{ (section 1 1)} = (200 - 3 \times 22) \times 10 = 1340 \text{ mm}^2 \text{ (governs)}$$

$$A_n \text{ (section 1221)} = [200 - 4 \times 22 + (2 \times 50^2) / (4 \times 30)] \times 10 = 1536.67 \text{ mm}^2$$

$$A_n \text{ (section 12321)} = [200 - 5 \times 22 + (4 \times 50^2) / (4 \times 30)] \times 10 = 1733.33 \text{ mm}^2$$

Factored design tension in members by

i) Yielding of gross section, $T_{dg} = f_y A_g / \gamma_{m0}$

$$= (250 \times 200 \times 10) / 1.10$$

$$= 454545.45 \text{ N}$$

$$= 454.4 \text{ kN}$$

Section 6.3.1

Section 6.2

ii) Rupture of net section, $T_{dn} = 0.9 f_u A_n / \gamma_{ml}$

$$= (0.9 \times 420 \times 1340) / 1.25$$
$$= 409752 \text{ N}$$
$$= 409.8 \text{ kN}$$

Section 6.3.1

Check for minimum edge and end distance:

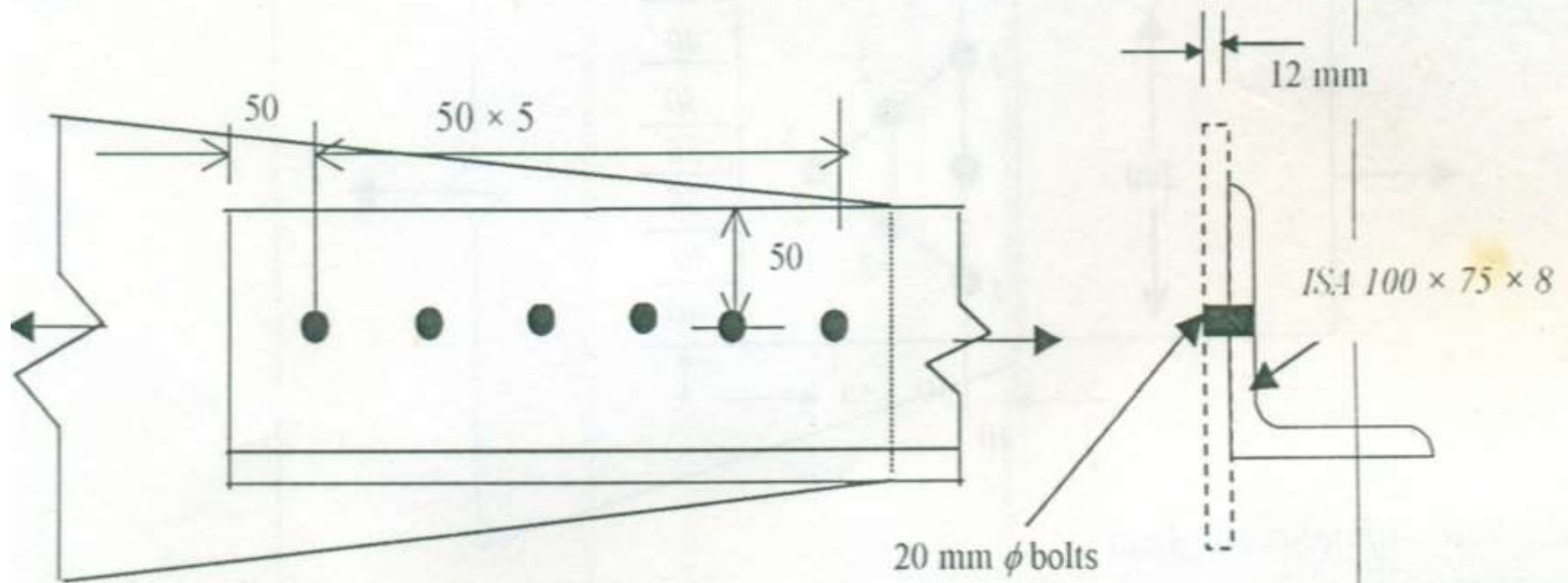
Provided minimum edge and end distance = 40 mm
which is greater than 32 mm (As per code)

The design tensile strength of the plate = 409.8 kN

The efficiency of the tension member, $= (409.8 \times 100) / (454.45)$
 $= 90.17\%$

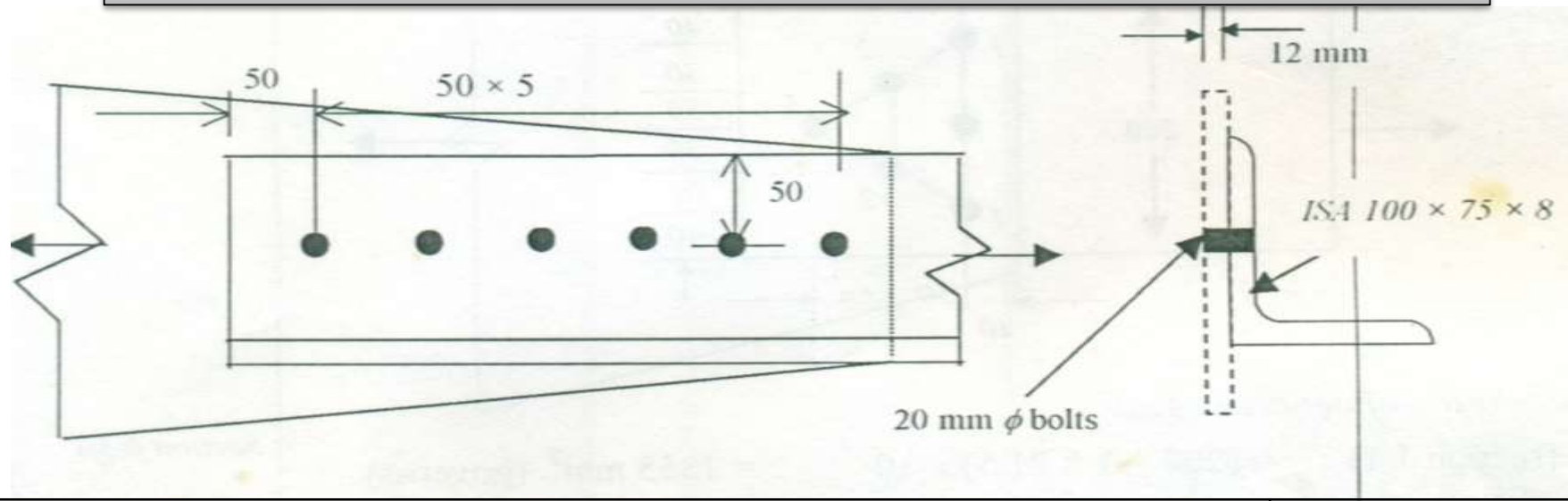
Tension member design: Example -2

A single unequal angle $100 \times 75 \times 8$ mm is connected to a 12 mm thick gusset plate at the ends with 6 nos. 20 mm diameter bolts to transfer tension. Determine the design tensile strength of the angle if the yield and the ultimate stress of the steel used are 250 MPa and 420 MPa. (a) if the gusset is connected to the 100 mm leg, (b) if the gusset is connected to the 75 mm leg.



Tension member design: Example -2

cont....



a) The 100mm leg bolted to the gusset :

$$A_{nc} = (100 - 8/2 - 22) \times 8 = 592 \text{ mm}^2.$$

$$A_{go} = (75 - 8/2) \times 8 = 568 \text{ mm}^2$$

$$A_g = ((100 - 8/2) + (75 - 8/2)) \times 8 = 1336 \text{ mm}^2$$

i) Strength as governed by yielding of gross section:

$$T_{dg} = A_g f_y / \gamma_{m0} = 1336 \times 250 / 1.10 = 303636 \text{ N} = 303.6 \text{ kN}$$

Section 6.2

ii) *Strength as governed by rupture of critical section:*

$$T_{dn} = 0.9 \times f_u \times A_{nc} / \gamma_{ml} + \beta \times A_{go} \times f_y / \gamma_{m0}$$

$$\beta = 1.4 - 0.035 (w/t) (f_u/f_y) (b/L)$$

$$= 1.4 - 0.035((75-4) / 8)(420/250)((46+71)/250) = 1.156$$

$$T_{dn} = [(0.9 \times 420 \times 596 / 1.25) + (1.156 \times 568 \times 250 / 1.10)]$$

$$= 329459 \text{ N} = 329.5 \text{ kN}$$

Section 6.3.3

iii) *Strength as governed by block shear:*

Section 6.4.2

$$T_{db} = [(0.577 \times f_y \times A_{vg} / \gamma_{m0}) + (0.9 \times f_u \times A_{tn} \times f_y / \gamma_{ml})]$$

$$= [(0.577 \times 250 \times (5 \times 50 + 50) \times 8 / 1.10) + (0.9 \times 420 \times (50 - 22/2) \times 8 / 1.25)]$$

$$= 337407 \text{ N} = 337.4 \text{ kN}$$

$$T_{db} [(0.52 \times f_u \times A_{vn} / \gamma_{ml}) + (A_{tg} f_y / \gamma_{m0})]$$

$$= [(0.52 \times 420 \times (5 \times 50 + 50 - 5.5 \times 22) \times 8 / 1.25) + (50 \times 250 \times 8 / 1.10)]$$

$$= 341108 \text{ N} = 341.1 \text{ kN}$$

The design tensile strength of angle member = 303.6 kN

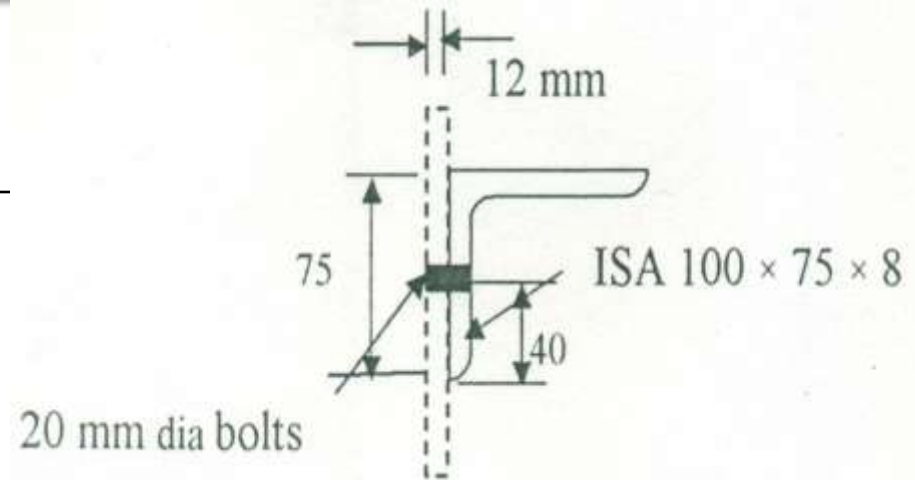
The efficiency of the tension member = 303.6×1000

$$\frac{\times 100}{(1336 \times 250 / 1.10)} = 100 \%$$

The 75 mm leg is bolted to the gusset:

$$A_n = (75 - 8/2 - 21.5) \times 8 = 396 \text{ mm}^2$$

$$A_o = (100 - 8/2) \times 8 = 768 \text{ mm}^2$$



i) Strength as governed by yielding of gross section:

$$T_{dg} = A_g f_y / \gamma_{m0} = 1336 \times 250 / 1.10 = 303636.36 \text{ N}$$

$$= 303.6 \text{ kN}$$

Section 6.2

ii) Strength as governed by tearing of net section:

$$T_{dn} = 0.9 \times f_u \times A_{nc} / \gamma_{m1} + \beta \times A_{go} \times f_y / \gamma_{m0}$$

Section 6.3.3

$$\beta = 1.4 - 0.035 (w/t) (f_u/f_y) (b/L)$$

$$= 1.4 - 0.035 (100/8) (420/250) (125/250) = 1.03$$

$$T_{dn} = 0.9 \times 420 \times 396 / 1.25 + 1.03 \times 768 \times 250 / 1.15$$

$$= 291715.6 \text{ N} = 291.7 \text{ kN}$$

Strength as governed by block shear:

$$\begin{aligned}
 T_{db} &= [(0.577 \times f_y \times A_{vg} / \gamma_{m0}) + (0.9 \times f_u \times A_{tn} \times f_y / \gamma_{ml})] \\
 &= [(0.577 \times 250 \times (5 \times 50 + 50) \times 8 / 1.10) \\
 &\quad + (0.9 \times 420 \times (40 - 22/2) \times 8 / 1.25)] \\
 &= 384884.07 \text{ N} \\
 &= 384.9 \text{ Kn}
 \end{aligned}$$

$$\begin{aligned}
 T_{db} &= [(0.52 \times f_u \times A_{vn} / \gamma_{ml}) + (A_{tg} \times f_y / \gamma_{m0})] \\
 &= [(0.52 \times 420 \times (5 \times 50 + 50 - 5.5 \times 21.5) \times 8 / 1.25) \\
 &\quad + (40 \times 250 \times 8 / 1.10)] \\
 &= 322926 \text{ N} = 322.9 \text{ kN}
 \end{aligned}$$

The design tensile strength of angle member = 290.4 kN

The efficiency of the tension member

$$\begin{aligned}
 &= (290.4 \times 1000 \times 100) / (1336 \times 250 / 1.10) \\
 &= 100 \%
 \end{aligned}$$

Section 6.4.2

3. Design a tension member to carry a load of 300 kN. The two angles placed back to back with long leg outstanding are desirable. The length of the member is 2.9 m.

Given Data:

$$T_u = 300 \text{ kN}, \quad \text{Length} = 2.9 \text{ m}$$

Solution

Area required from the consideration of yielding

$$A_g = \frac{T_u}{(f_y / \gamma_{mo})}$$

(Refer table-5 of IS 800:2007)

Assume γ_{mo} (partial safety factor) = 1.1

Assume γ_{ml} (partial safety factor) = 1.25

$$A_g = \frac{300 \times 1000}{250 / 1.1} = 1320 \text{ mm}^2$$

Try 2 ISA 75 X 50 X 8mm thick which has gross area = 2 x 938
= 1876 mm²

Strength of 20 mm black bolts:

$$(a) \text{ In double shear } = \left[\left[\frac{\pi}{4} \times 20^2 + 0.78 \times 20^2 \right] \times \frac{\pi}{4} \times 20^2 \right] \times \frac{400}{\sqrt{3}}$$

$$\times \frac{400}{\sqrt{3}} \times \frac{1}{1.25} = 103314 \text{ N}$$

(b) Strength in bearing:

Taking $e = 40 \text{ mm}$, $p = 60 \text{ mm}$

Where , k_b is smaller of $\frac{e}{3 d_o}$, $\left(\frac{p}{3 d_o} - 0.25\right)$, $\frac{f_{ub}}{f_u}$, 1.0 ;

$$k_b \text{ is smaller of } \frac{40}{3 \times 22} , \left(\frac{60}{3 \times 22} - 0.25\right) , \frac{400}{410} , 1.0;$$

k_b is smaller of 0.606, 0.909, 0.97, 1.0.

Therefore $k_b = 0.606$

$$= 2.5 \cdot k_b \cdot d \cdot f_u$$

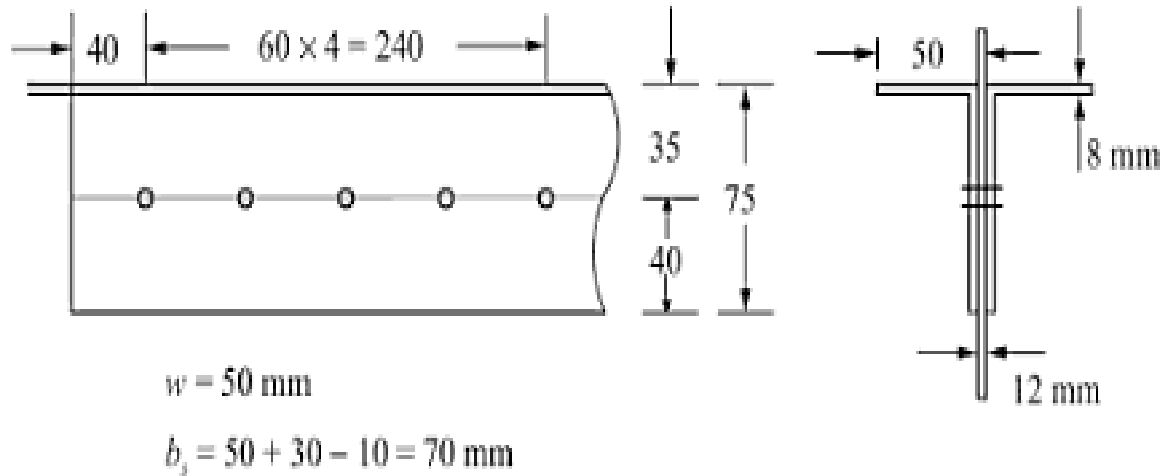
$$V_{npb} = 2.5 \times 20 \times 8 \times 400 \\ = 96960 \text{ kN.}$$

The design bearing strength of the bolt,

$$V_{dpb} = \frac{V_{npb}}{\gamma_{mb}} = \frac{96960}{1.25} = 77568 \text{ N}$$

$$\text{Number of bolts required} = \frac{300 \times 1000}{77568} = 0.39$$

Provide 5 bolts in a row as shown in fig



Checking the design

(a) Strength against yielding,

$$\begin{aligned}
 T_{dg} &= \frac{A_g f_y}{\gamma_{mo}} \\
 &= \frac{1876 \times 250}{1.1} \\
 &= 426364 \text{ N} > 300 \times 1000
 \end{aligned}$$

(b) Strength of plate in rupture , $T_{dn} = \frac{0.9 X A_{nc} f_u}{\gamma_{ml}} + \frac{\beta A_{go} f_y}{\gamma_{mo}}$

Area of connected leg, $A_{nc} = 2\left(75 - 22 - \frac{8}{2}\right) \times 8$
 $= 784 \text{ mm}^2$

Area of outstanding leg, $A_{go} = 2\left(50 - \frac{8}{2}\right) \times 8$
 $= 736 \text{ mm}^2$

$$\beta = 1.4 - 0.076 \times \frac{w}{t} \times \frac{f_y}{f_u} \times \frac{b_s}{L_c}$$

$$= 1.4 - 0.076 \times \frac{50}{8} \times \frac{250}{410} \times \frac{77}{240}$$

$$= 1.307$$

$$\begin{aligned} \text{Therefore, } T_{dn} &= \frac{0.9 \times A_{nc} f_u}{\gamma_{ml}} + \frac{\beta A_{go} f_y}{\gamma_{mo}} \\ &= \frac{0.9 \times 410 \times 784}{1.25} + \frac{1.307 \times 736 \times 250}{1.1} \\ &= 450062 > 300000 \end{aligned}$$

(c) Strength against block shear failure

Per angle:

$$A_{vg} = (40 + 60 \times 4) \times 8 = 2240 \text{ mm}^2$$

$$A_{vn} = (40 + 60 \times 4 - 4.5 \times 22) \times 8 = 1448 \text{ mm}^2$$

$$A_{tg} = (75 - 35) \times 8 = 320 \text{ mm}^2$$

$$A_{tn} = (75 - 35 - 0.5 \times 22) \times 8 = 232 \text{ mm}^2$$

Strength against block failure of each angle is the smaller of the following two values:

As per IS 800 – 2007 clause 6.4.1,

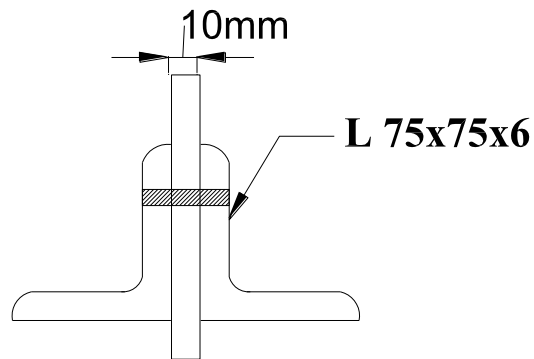
$$\begin{aligned} \text{i) } T_{db} &= \frac{A_{vg} f_y}{\sqrt{3} \gamma_{mo}} + \frac{0.9 A_{tn} f_u}{\gamma_{ml}} \\ &= \frac{2240 \times 250}{\sqrt{3} \times 1.1} + \frac{0.9 \times 232 \times 410}{1.25} \\ &= 362410 \text{ N} \end{aligned}$$

$$\begin{aligned}
 \text{ii) } T_{db} &= \frac{0.9 A_{vn} f_u}{\sqrt{3} \gamma_{ml}} + \frac{A_{tg} f_y}{\gamma_{mo}} \\
 &= \frac{0.9 \times 1448 \times 410}{\sqrt{3} \times 1.25} + \frac{320 \times 250}{1.1} \\
 &= 319515 \text{ N}
 \end{aligned}$$

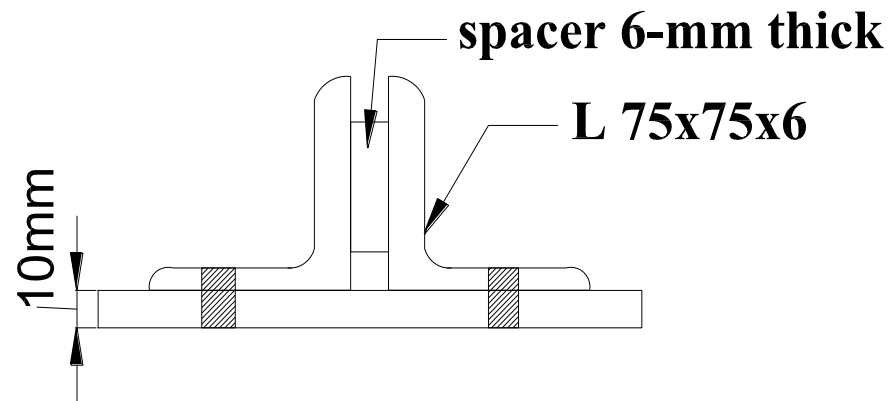
Strength of two angles against block failure = 2 x 319515 N
 = 639030 > 300000 N O.K.

Hence use 2 ISA 75 X50X8 mm with 5 bolts of 20 mm diameter.

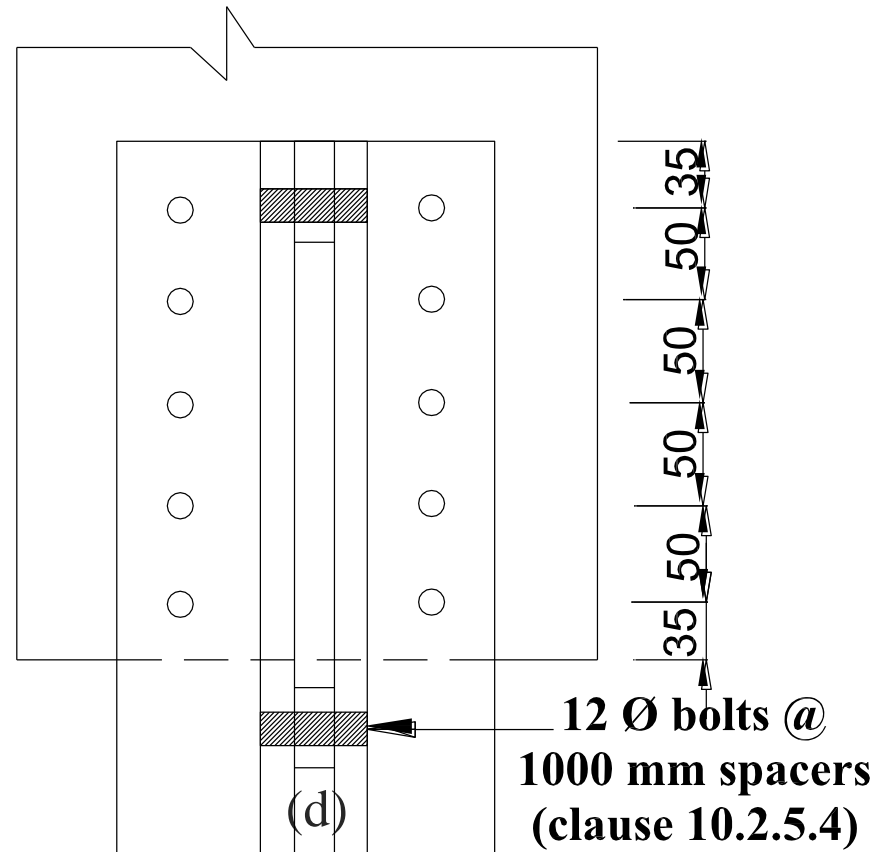
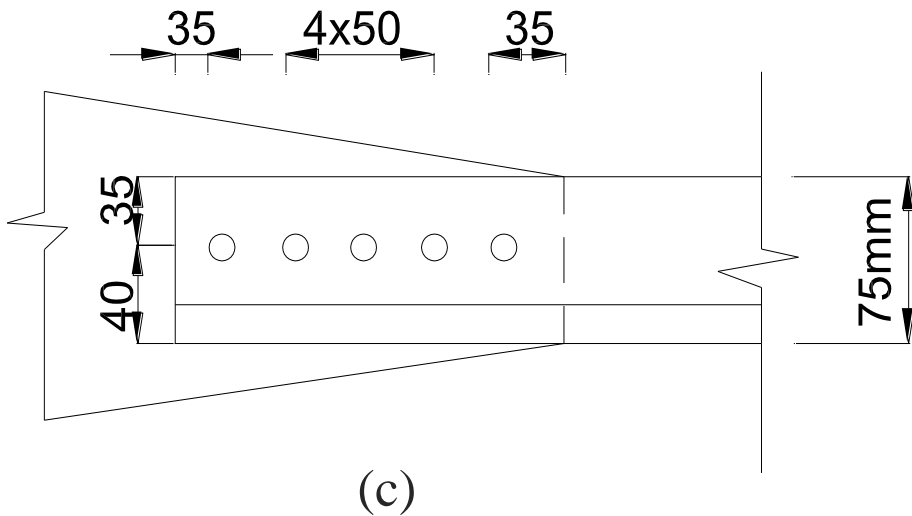
4. A tie member in a bracing system consists of two angles 75x75x6 bolted to a 10mm gusset one on each side using a single row of bolts [see given fig.A] and tack bolted. Determine the tensile capacity of the member and the number of bolts required to develop full capacity of the member. What will be the capacity if the angles are connected on the same side of gusset plate And tack bolted [see fig B]. What is the effects on tensile strength if the members are not tack bolted?



Connected to gusset
one on each side –(a)



Connected to the same side of
of the gusset – (b)



Solution:

(a) Two angles connected to the opposite side of the gusset as in Fig (a)

(i) Design strength due to yielding of gross section

$$T_{dg} = f_y (A_g / \gamma_{m0})$$

$$A_g = 866 \text{ mm}^2 \text{ (for single angle)}$$

$$T_{dg} = 250 \times 2 \times (866 / 1.10) \times 10^{-3}$$

$$T_{dg} = 393.64 \text{ kN.}$$

(ii) The design strength governed by tearing at net section

$$T_{dn} = \alpha A_n (f_u / \gamma_{m1})$$

Assume a single line of four numbers of 20mm-diameter bolts

($\alpha = 0.8$)

$$A_n = [(75 - 6/2 - 22)6 + (75 - 6/2)6]2$$

$$A_n = (300 + 432)2 = 1464 \text{ mm}^2$$

$$T_{dn} = (0.8 \times 1464 \times 410 / 1.25) = 384.15 \text{ kN}$$

Therefore, Tensile capacity = 384.15 kN

Design of bolts

Choose edge distance = 35mm

capacity of bolt in double shear (table 5.9)

$$= 2 \times 45.3 = 90.6 \text{ kN.}$$

Bearing capacity of bolt does not govern as per table 5.9

Hence,

Strength of a single bolt = 90.6 kN.

Provide 5 bolts then,

Total strength of the bolts = $5 \times 90.6 = 453 \text{ kN} > 384.15 \text{ kN}$.

Hence the connection is safe.

Minimum spacing $= 2.5t = 2.5 \times 20 = 50\text{mm}$.

Hence, provide a spacing of 50mm.

The arrangements of the bolts are shown in fig (c).

Check for block shear strength :(clause 6.4)

Block shear strength T_{db} of connection will be taken as

$$T_{db1} = [(A_{vg} f_y / \sqrt{3} \gamma_{m0}) + (0.9 A_{tn} f_u / \gamma_{m1})]$$

or

$$T_{db2} = [(0.9 f_u A_{vn} / \sqrt{3} \gamma_{m1}) + (f_y A_{tg} / \gamma_{m0})]$$

Whichever is smaller

$$A_{vg} = (4 \times 50 + 35)6 = 1410\text{mm}^2$$

$$A_{vn} = (4 \times 50 + 35)6 = 1410\text{mm}^2$$

$$A_{tn} = (35.0 - 22/2)6 = 144\text{mm}^2$$

$$A_{tg} = (35 \times 6) = 210\text{mm}^2$$

$$T_{db1} = \left\{ \left[\frac{1410 \times 250}{\sqrt{3} \times 1.10} \right] + \left[0.9 \times 144 \times 410 / 1.25 \right] \right\} \times 10^{-3}$$
$$= 227.5\text{kN}.$$

$$T_{db2} = \left\{ \left[\frac{0.9 \times 410 \times 816}{\sqrt{3} \times 1.25} \right] + \left[\frac{250 \times 210}{1.10} \right] \right\} \times 10^{-3}$$
$$= 186.8\text{kN}.$$

For double angle ,

$$\text{Block shear strength} = 2 \times 186.8 = 373.6 \text{ kN}.$$

Therefore,

$$\text{Tensile capacity} = 373.6 \text{ kN. (least of } \\ 393.64\text{kN, } 384.14\text{kN, } 373.6\text{kN.)}$$

(b) Two angles connected to the same side of the gusset plate (fig –b)

(i) Design strength due to yielding of the gross section
= 394.64kN.

(ii) Design strength governed by tearing at the net section
= 384.14 kN.

Assuming 10 bolts of 20mm diameter ,five bolts in each connected leg

Capacity of an M20 bolts in single shear = 45.3kN.

Total strength of bolts = $10 \times 45.3 = 453 \text{ kN} > 394.64\text{kN}$.

Hence the connection is safe.

The arrangements of bolts is shown in fig (d).Since it is similar to the arrangement shown in fig (c),the block shear strength will

the same i.e., 373.6kN.

Hence the tensile capacity =373.6kN.

The tensile capacity of both the arrangements (angles connected on the same side and connected to the opposite side of gusset) are same as per the code though the load application is eccentric in this case .Moreover the number of bolts are ten whereas in case (a) we used only five bolts since the bolts were in double shear.

(c) If the angles are not tack bolted ,they behave as single angles connected to gusset plate

In this case also the tensile capacity will be the same and we have to use ten M20 bolts.This fact is confirmed by the test and

- FEM results stating that ‘ the net section strength of double angles on opposite sides of the gusset and tack connected adequately over the length is nearly the same as that of two single angles acting individually.
- current design provisions indicating greater efficiency of such double angles are not supported by the tests and FEM results.

5. Select a suitable angle section to carry a factored tensile force of 210 kN assuming a single row of M20 bolts and assuming design strength as $f_y = 250 \text{ N/mm}^2$.

Solution:

Approximate required area = $1.1 \times 210 \times 10^3 / 250 = 924 \text{ mm}^2$

Choose 65 x 65 x 8 angle with $A = 976 \text{ mm}^2$

Strength governed by yielding = $[976 \times 250 / 1.1] \times 10^{-3}$
= 221.81 kN

A_{nc} = area of connected leg = $(65 - 4 - 22) \times 8 = 312 \text{ mm}^2$

A_{go} = $(65 - 4) \times 8 = 488 \text{ mm}^2$

Required number of M20 bolts (Table 5.9) = $170 / 45.3 = 3.75$

Provide four bolts at the pitch of 60 mm.

Strength governed by rupture of critical section

$$T_{dn} = 0.9f_u A_{nc} / \gamma_{m1} + \beta A_{go} f_y / \gamma_{mo}$$

$$\beta = 1.4 - 0.076 \times (65/8)(250/410)(61 + 35)/(3 \times 60)$$
$$= 1.199$$

$$T_{dn} = (0.9 \times 410 \times 312 / 1.25 + 1.199 \times 488 \times 250 / 1.10) \times 10^{-3}$$
$$= 225.08 \text{ kN.}$$

Alternatively,

$$T_{dn} = \alpha A_n f_u / \gamma_{m1}$$
$$= [0.8 \times (312 + 488) \times 410 / 1.25] \times 10^{-3}$$
$$= 209.92 \text{ kN.}$$

Strength governed by block shear

Assuming an edge distance of 40mm.

$$A_{vg} = 8 \times (3 \times 60 + 40) = 1760 \text{ mm}^2$$

$$A_{vg} = 8 \times (3 \times 60 + 40 - 3.5 \times 22) = 1144 \text{ mm}^2$$

$$A_{tg} = 8 \times 35 = 208 \text{ mm}^2$$

$$A_{tn} = 8 \times (35 - 0.5 \times 22) = 192 \text{ mm}^2$$

$$T_{db1} = [1760 \times 250 / (\sqrt{3} \times 1.1) + 0.9 \times 410 \times 192 / 1.25] \times 10^{-3}$$
$$= 287.61 \text{ kN.}$$

$$T_{db1} = [0.9 \times 410 \times 1144 / (\sqrt{3} \times 1.25) + 250 \times 280 / 1.1] \times 10^{-3}$$
$$= 258.61 \text{ kN.}$$

Tension capacity of the angle = 209.92 ~ 210 kN.

Hence the angle is safe

Thank You

DESIGN OF STEEL STRUCTURES

by

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Professor,

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Annamalai University**

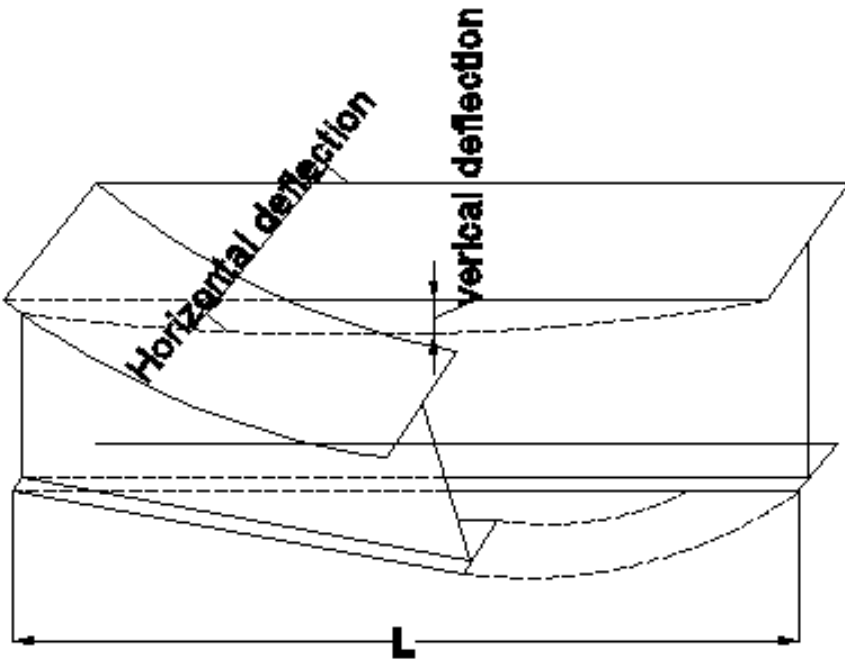
BEAMS

- One of the frequently used structural members is a beam whose main function is to transfer load principally by means of flexural or bending action.
- In a structural framework, it forms the main horizontal member spanning between adjacent columns or as a secondary member transmitting floor loading to the main beams.
- Normally only bending effects are predominant in a beam except in special cases such as crane girders, where effects of torsion in addition to bending have to be specifically considered.

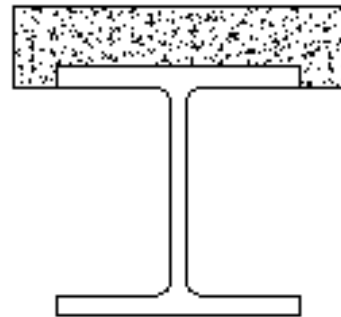
LATERALLY SUPPORTED BEAM

- When the lateral support to the compression flange is adequate, the lateral buckling of the beam is prevented and the section flexural strength of the beam can be developed.
- The strength of I-sections depends upon the width to thickness ratio of the compression flange.
- When the width to thickness ratio is sufficiently small, the beam can be fully plastified and reach the plastic moment, such sections are classified as compact sections.
- However provided the section can also sustain the moment during the additional plastic hinge rotation till the failure mechanism is formed. Such sections are referred to as plastic sections.

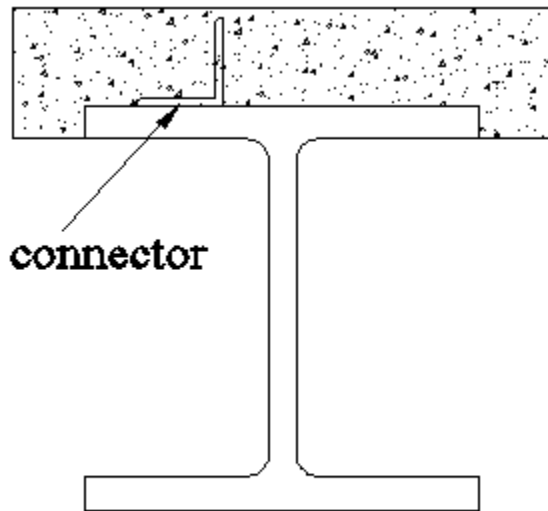
LATERALLY SUPPORTED BEAM



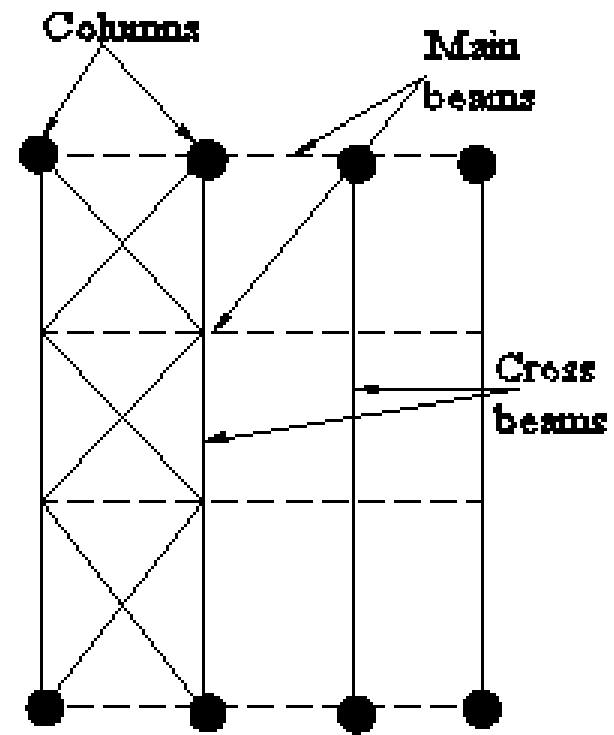
(a) Buckling of compression flange



(b)



(c)

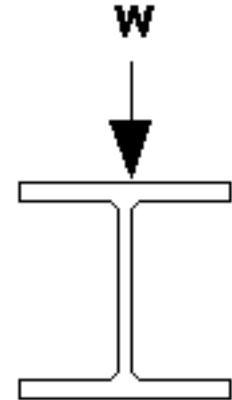
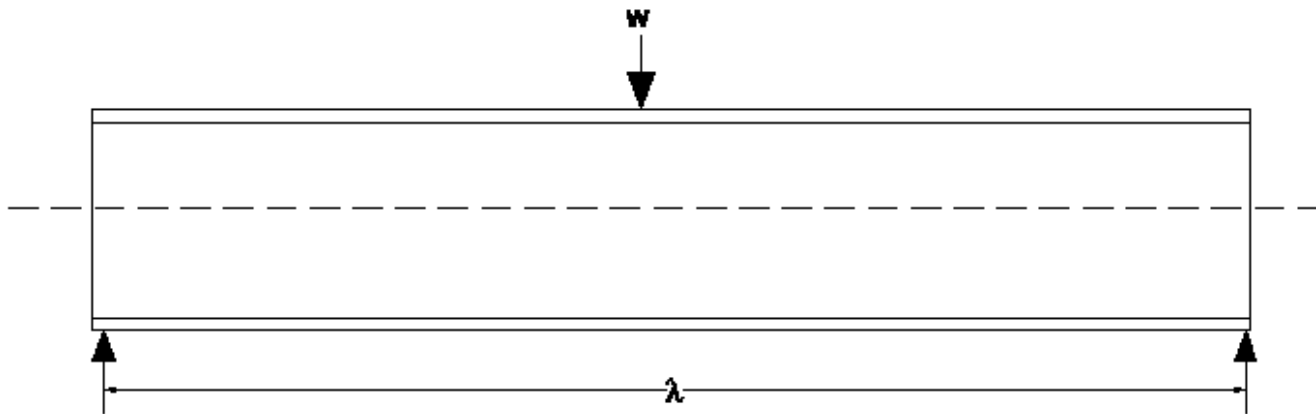


(d)

- When the compression flange width to thickness ratio is larger, the compression flange may buckle locally before the complete plastification of the section occurs and the plastic moment is reached.
- Such sections are referred to as non-compact sections.
- When the width to thickness ratio of the compression flange is sufficiently large, local buckling of compression flange may occur even before extreme fibre yields.
- Such sections are referred to as slender sections.

LATERALLY UNSUPPORTED BEAMS

- Under increasing transverse loads, a beam should attain its full plastic moment capacity.



Two important assumptions have been made therein to achieve the ideal beam behaviour.

They are:

- The compression flange of the beam is restrained from moving laterally; and
- Any form of local buckling is prevented.

1. Design a continuous beam of spans 4.9 m, 6 m, and 4.9 m carrying a uniformly distributed load of **32.5 kN/m** and the beam is laterally supported.

Factored load calculation

Factored uniformly distributed load = $1.5 \times 32.5 = 48.75$ kN/m

The bending moment and shear force distribution are shown below

Maximum bending moment = 146.25 kN m

Maximum shear force = $146.25 + 146.25 = 292.5$ kN

Plastic section modulus required

$$Z_p = \frac{M \times \gamma_{mo}}{f_y} = \frac{146.25 \times 10^6 \times 1.10}{250} = 643.5 \times 10^3 \text{ mm}^3$$

Selection of suitable section

Choose a trial section of ISLB 350 @0.495 kN/m.

Overall depth (h) = 350 mm

Width of flange (b) = 165 mm

Thickness of flange (t_f) = 11.4 mm

Depth of web (d) = $h - 2(t_f + R) = 350 - 2(11.4 + 16) = 295.2$ mm

Thickness of web (t_w) = 7.4 mm

Moment of inertia about major axis $I_x = 13158.3 \times 10^4$ mm⁴

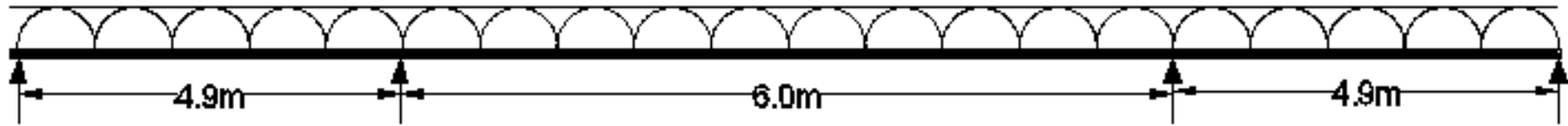
Elastic section modulus (Z_e) = 751.9×10^3 mm³

Plastic section modulus (Z_p) = 851.11×10^3 mm³

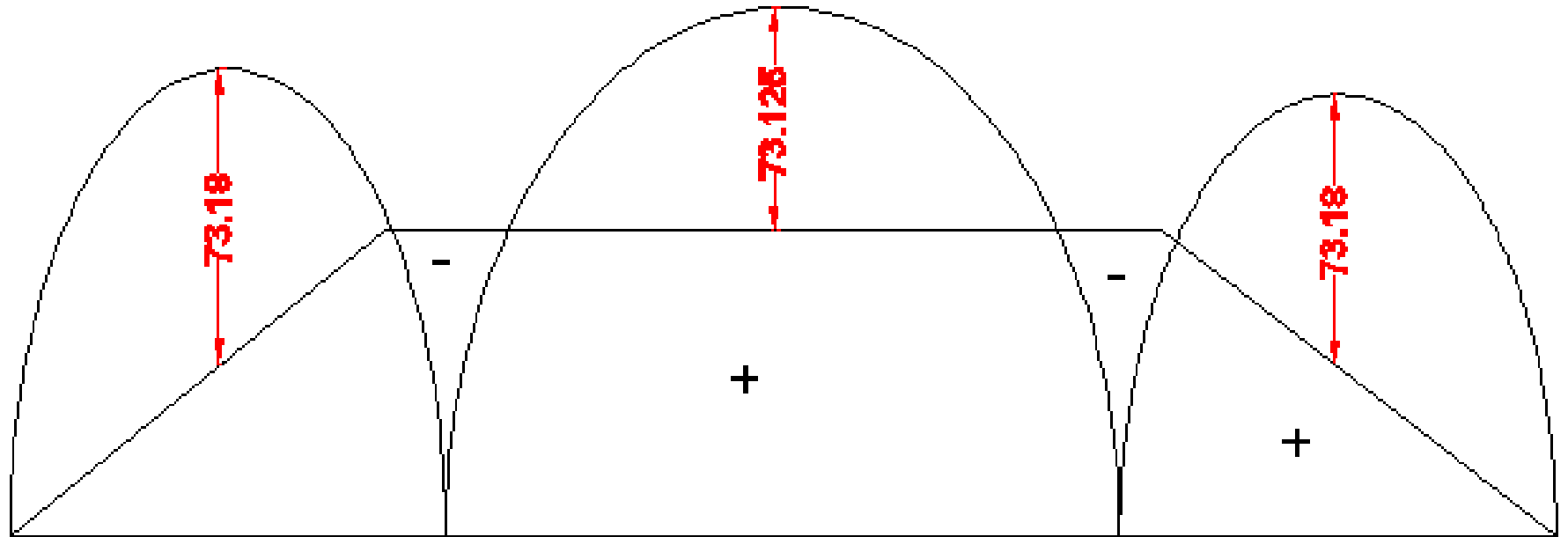
Section classification

$$\frac{b}{t_f} = \frac{82.5}{11.4}$$

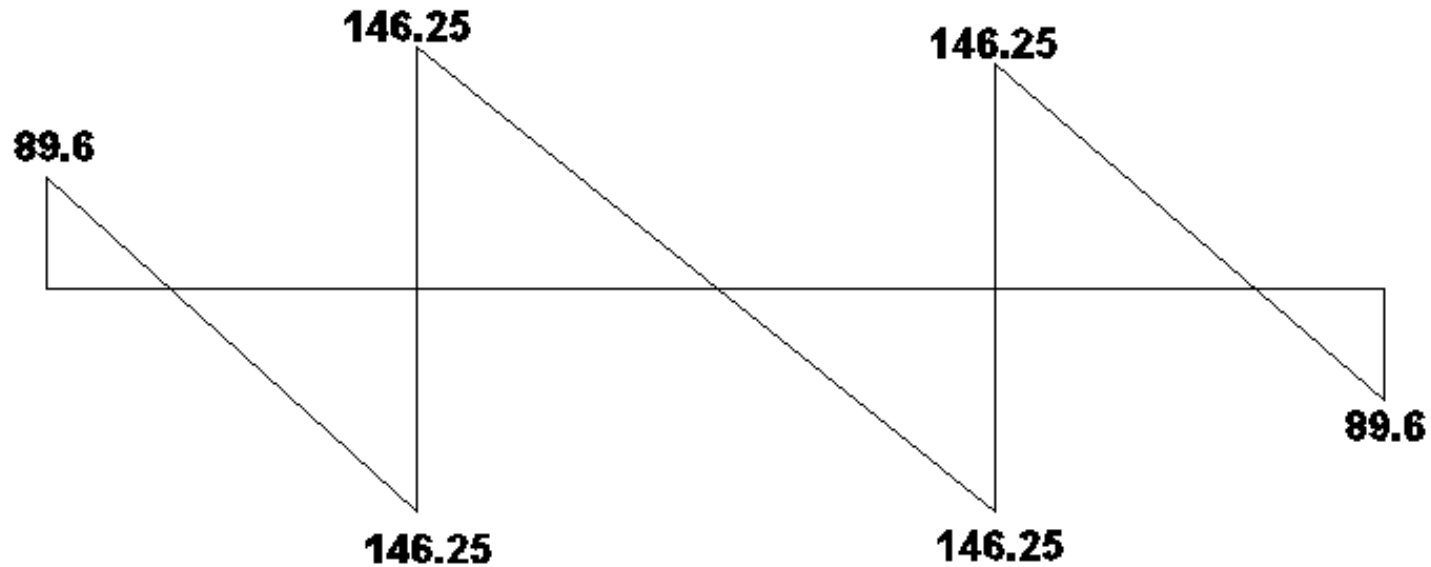
48.75 kN/m



BEAM LOADING (a)



BENDING MOMENT DIAGRAM (b)



SF DIAGRAM (c)

$$\frac{b}{t_f} = \frac{295.2}{7.4} = 39.9 < 84$$

Hence the section is plastic.

Check for shear capacity of section

$$V_d = \frac{f_y}{m_o \times \sqrt{3}} \times h \times t_w = \frac{250}{1.1 \times \sqrt{3}} \times 350 \times 7.4 = 340 \text{ kN}$$

$$0.6 v_d = 204 \text{ kN} < 292.5 \text{ kN}$$

This shows a high shear condition.

Check for moment capacity of the section [Eqn 6.8(a)]

$$M_{dv} = M_d - \beta (M_d - M_{fd}) \leq 1.09 \times Z_e \times f_y$$

where M_{fd} is the plastic design strength of the area of cross section excluding the shear area.

$$\beta = \left[2 \times \left(\frac{v}{v_d} \right) \times 1 \right]^2 = \left[2 \times \left(\frac{292.5}{340} \right) \times 1 \right]^2$$

Calculation of section modulus of flange

$$Z_{fd} = Z_p - A_w y_w$$

$$= 851.11 \times 10^3 - \left(350 \times 7.4 \times \frac{350}{4} \right)$$

$$= 624.485 \times 10^3 \text{ mm}^3$$

$$\begin{aligned}
 \text{Therefore, } M_{fd} &= \frac{Z_{fd} \times f_y}{\gamma_{mo}} \\
 &= \frac{624.485 \times 10^3 \times 250}{1.10} \\
 &= 141.93 \text{ kNm}
 \end{aligned}$$

Moment capacity of the section

$$\begin{aligned}
 M_d &= \frac{Z_p \times f_y}{\gamma_{mo}} = \frac{851.11 \times 10^3 \times 250}{1.10} \\
 &= 193.43 \text{ kNm}
 \end{aligned}$$

therefore, $M_{dv} = 193.43 - 0.52(193.43 - 141.93)$

$$= 165.65 \text{ kN m} < \frac{1.2 \times Z_e \times f_y}{\gamma_{mo}} = \frac{1.2 \times 751.9 \times 10^3 \times 250}{1.10}$$

$$= 205.06 \text{ kN m} > 146.25 \text{ kN m}$$

Hence the section is safe.

2. Design a laterally unrestrained beam to carry a uniformly distributed load of 30 kN/m. The beam is unsupported for a length of 3 m and is simply placed on longitudinal beams at its ends.

Calculation of load

Factored load = $1.5 \times 30 = 45$ kN/m

Calculation of bending moment and shear force

$$\text{BM} = \frac{wl^2}{8} = \frac{45 \times 3^2}{8} = 50.625 \text{ kN.m}$$

$$\text{SF} = \frac{wl}{2} = \frac{45 \times 3}{2} = 67.5 \text{ kN}$$

Initialization of section

Assume $\lambda = 100$; $\frac{h}{t_f} = 25$ and hence from

table 14, $f_{cr,b} = 291.31 \text{ N/mm}^2$

$$\lambda_{LT} = \frac{\sqrt{f_y}}{\sqrt{f_{crb}}} = \frac{\sqrt{250}}{\sqrt{291.31}} = 0.926$$

$$\begin{aligned}\Phi_{LT} &= 0.5[1 + \alpha_{LT}(\lambda_{LT} - 0.2) + \lambda_{LT}^2] \\ &= 0.5[1 + 0.21(0.926 - 0.2) + 0.926^2] = 1.005\end{aligned}$$

$$\begin{aligned}\chi_{LT} &= \frac{1}{\Phi_{LT} + [\Phi_{LT}^2 - \lambda_{LT}^2]^{0.5}} \leq 1.0 \\ &= \frac{1}{1.005 + [1.005^2 - 0.926^2]^{0.5}} = 0.716 \leq 1.0\end{aligned}$$

$$f_{bd} = \frac{\chi_{LT} f_y}{\gamma_{mo}} = \frac{0.716 \times 250}{1.10} = 162.7 \text{ N/mm}^2$$

$$\begin{aligned}\text{Therefore required z of section} &= \frac{50.625 \times 10^6}{162.7} \\ &= 311.1 \times 10^3 \text{ mm}^3\end{aligned}$$

Choose a section of ISMB 225 @ 0.3 12 kN/m.

Overall depth (D) = 225 mm

Width of flange (b_f) = 110 mm

Thickness of flange (t_f) = 11.8 mm

Thickness of web (t_w) = 6.5 mm

Depth of web (d) = $D - 2(t_f + R) = 225 - 2(11.8 + 12) = 177.4 \text{ mm}$

Moment of inertia about major axis $I_{zz} = 3440 \times 10^4 \text{ mm}^4$

Moment of Inertia about minor axis $I_{yy} = 218 \times 10^4 \text{ mm}^4$

Elastic section modulus (Z_{ez}) = $305.9 \times 10^3 \text{ mm}^3$

Plastic section modulus (Z_{ey}) = $348.27 \times 10^3 \text{ mm}^3$

Minimum radius of gyration (r_y) = 18.6 mm

Section classification

Outstand of compression flange = $(110/2)/11.8 = 4.66 < 9.4$

Web with neutral axis at mid depth = $177.4/6.5 = 27.3 < 84$

Therefore the section is plastic.

Calculation of lateral-torsional buckling moment

$$M_{cr} = \sqrt{\frac{\pi^2 EI_y}{(KL)^2} \left(GI_t + \frac{\pi^2 EI_w}{(KL)^2} \right)} \quad (\text{from clause 8.2.2.1)/p-54}$$

$$G = \frac{E}{2(1 + \mu)} = \frac{2 \times 10^5}{2(1 + 0.3)} = 76.923 \times 10^3 \text{ N/mm}^2$$

$$I_t = \sum \frac{b_i t_i^3}{3} = \frac{2 \times 110 \times 11.8^3}{3} + \frac{(225 - 2 \times 11.8) \times 6.5^3}{3}$$

$$= 138.926 \times 10^3 \text{mm}^3$$

$$I_w = (1-\beta_f) \beta_f I_y h_f^2$$

$$\beta_f = \frac{I_{fc}}{I_{fc} + I_{ft}} = 0.5$$

$$h_f = 225 - 11.8 = 213.2 \text{ mm}$$

$$I_w = (1-0.5) \times 0.5 \times 218 \times 10^4 \times 213.2^2 = 24.77 \times 10^9 \text{mm}^6$$

$$M_{cr} = \sqrt{\frac{\pi^2 \times 2 \times 10^5 \times 218 \times 10^4}{3000^2} (76.923 \times 10^3 + 138.926 \times 10^3)}$$

$$+ \frac{\pi^2 \times 2 \times 10^5 \times 24.77 \times 10^9}{2}$$

$$= 87.79 \text{kNm}$$

$$\lambda_{LT} = \sqrt{\frac{Z_p f_y}{M_{cr}}} = \sqrt{\frac{348.27 \times 10^3 \times 250}{87.79 \times 10^6}} = 0.9959$$

$$\Phi_{LT} = 0.5[1 + \alpha_{LT}(\lambda_{LT} - 0.2) + \lambda_{LT}^2]$$

$$\alpha_{LT} = 0.5[1 + \alpha_{LT}(\lambda_{LT} - 0.2) + \lambda_{LT}^2]$$

$$\alpha_{LT} = 0.21$$

$$\Phi_{LT} = 0.5[1 + \alpha_{LT}(\lambda_{LT} - 0.2) + \lambda_{LT}^2]$$

$$\chi_{LT} = \frac{1}{[\Phi_{LT}^2 + \lambda_{LT}^2]^{0.5}} = 0.6685 \leq 1.0$$

$$f_{bd} = \frac{\chi_{LT} f_y}{\gamma_{mo}} = \frac{0.6685 \times 250}{1.10} = 151.93 \text{ N/mm}^2$$

$$M_d = Z_p f_{bd} = 348.27 \times 10^3 \times 151.93 = 52.91 \text{ kNm} \\ > 50.625 \text{ kNm}$$

Calculation of shear capacity of section

$$V_d = \frac{f_y}{\gamma_{mo} \sqrt{3}} \times D \times t_w = \frac{250}{1.10 \times \sqrt{3}} \times 225 \times 6.5 \\ = 191. \text{ kN}$$

$$0.6 V_d = 115 \text{ kN} > 67.5 \text{ kN}$$

Calculation of deflection

$$\delta_b = \frac{5wl^4}{384EI}, \quad w = 30 \text{ kN/m}$$

$$\delta_b = \frac{5 \times 30 \times 3000^4}{384 \times 2 \times 10^5 \times 3440 \times 10^4} = 4.6mm$$

$$\text{Allowable deflection} = \frac{l}{300} = \frac{3000}{300} = 10mm$$

Hence the section is safe against deflection.

Check for web buckling:

Assuming that longitudinal beams are of the same size,

$$A_b = (b_1 + n_1)t_w = 4.6mm$$

$$b_1 = \frac{(b_f - t_w)}{2} = \frac{110 - 6.5}{2} = 51.75mm$$

$$n_1 = \frac{D}{2} = \frac{225}{2} = 112.5mm$$

$$A_b = (51.75+112.5) \times 6.5 = 1067.6mm^2$$

$$r_{min} = \sqrt{\frac{I}{A}} = \sqrt{\frac{1184}{336.4}} = 1.88mm$$

$$\lambda = \frac{l_{eff}}{r_{min}} = \frac{0.7 \times 177.4}{1.88} = 66.18$$

therefore, $f_{cd} = 158.36N/mm^2$ (from table 9c of the code)

$$\begin{aligned} \text{Strength of the section against web buckling} &= 158.36 \times 1067.6 \\ &= 169.07 \text{ kN} > \mathbf{67.5 \text{ kN}} \end{aligned}$$

Check for web bearing:

$$F_w = (b_1 + n_2)t_w f_y / \gamma_{mo}$$

$$b_1 = 51.75 \text{ mm}$$

$$n_2 = 2.5(t_f + R) = 2.5(11.8 + 12) = 59.5 \text{ mm}$$

$$F_w = (51.75 + 59.5) \times 6.5 \times 250 / (1.10 \times 10^3) = 164.35 \text{ kN} > 67.5 \text{ kN}$$

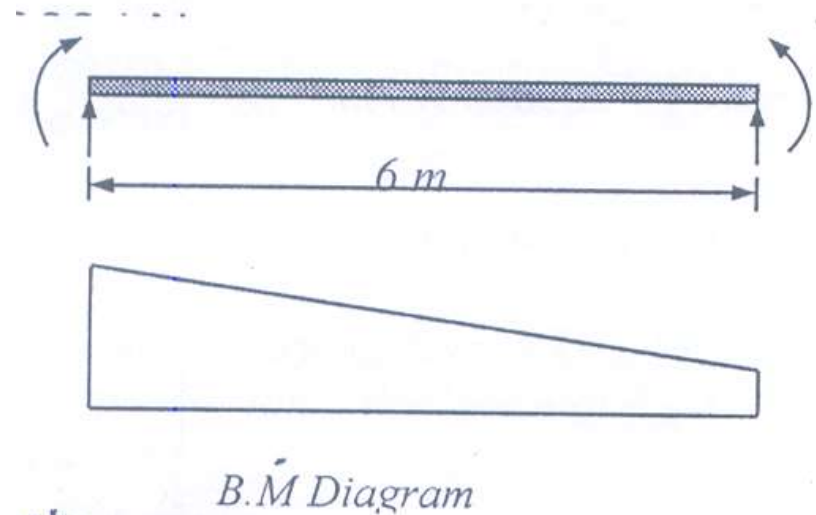
Hence the section is safe against web bearing.

PROBLEMS

3. A simply supported beam of span 6m is subjected to end moments of 202 kN.m (clockwise) and 112 kN.m (anticlockwise) under factored applied loading. Check whether ISMB-450 is safe with regard to lateral buckling.

Design check

For the end conditions given, it is assumed that the beam is simply supported in a vertical plane, and at the ends the beam is fully restrained against lateral deflection and twist with



no rotational restraints in plan at its ends.

Section classification of ISMB 450

The properties of the section are:

Depth, $h = 450\text{mm}$

Width, $b = 150\text{ mm}$

Web thickness, $t_w = 9.4\text{ mm}$

Flange thickness, $t_f = 17.4\text{ mm}$

$I_y = 834 \times 10^4\text{ mm}^4$

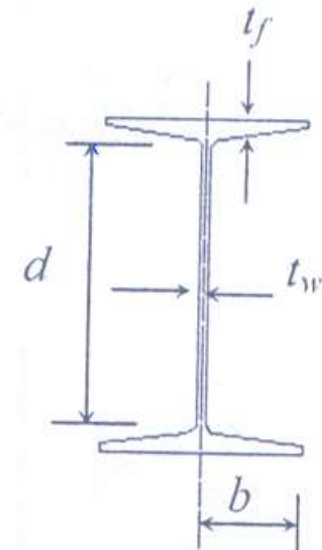
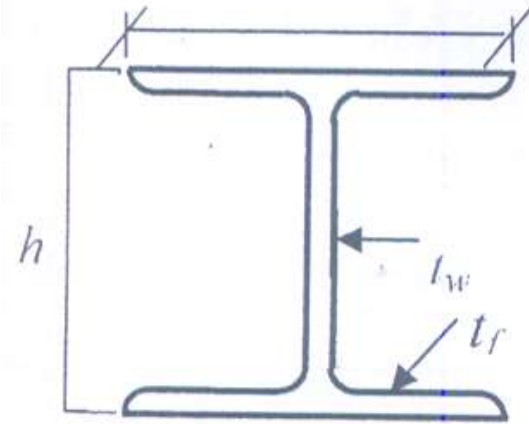
Depth between fillets, $d = 379.2\text{ mm}$

Radius of gyration about minor axis,

$r_y = 30.1\text{ mm}$

Plastic modulus about major axis,

$z_p = 1533.36 \times 10^3\text{ mm}^3$



Rolled Steel Beams

Assume $f_y = 250 \text{ N/mm}^2$, $E = 200000 \text{ N/mm}^2$, $\gamma_m = 1.10$

Type of section

Flange criterion:

$$b = B/2 = 150/2 = 75\text{mm}$$

$$b/t_f = 75.0 / 17.4 = 4.31$$

$$b/t_f = 9.4\varepsilon \quad \text{where } \varepsilon = \sqrt{250/f_y}$$

Hence , O.K

Web criterion:

$$d/t_w = 379.2/9.4 = 40.3$$

$$d/t_w < 84 \varepsilon$$

Hence, O.K

Since, $b/t_f = 9.4\varepsilon < d/t_w < 84 \varepsilon$,the section is classified as

'plastic'

*Table 3.1(section 3.7.2 of
I.S 800)*

Check for lateral torsional buckling :

Check for slenderness ratio:

Effective length criteria:

With ends of compression flanges fully restrained for torsion at support but both the flanges are not restrained against

warping, Effective length of simply supported beam, $L_{LT} = 1.0 L$

Where L is the span of the beam. (*Table 8.3 of I.S.800*)

$$\text{Hence, } L_{LT} = 1.0 \times 6.0 \text{ M} = 6000\text{mm}, \quad L_{LT}/r = 6000/30.1 \\ = 199.33$$

Since the moment is varying from 155 k-Nm to 86 k-Nm, there will be moment gradient. So for calculation f_{bd} , critical moment, M_{cr} is to be calculated

Now, critical moment

$$M_{cr} = C_1 \frac{\pi^2 EI}{(KL)^2} \left\{ \left[\left(\frac{K}{K_w} \right)^2 \frac{I_w}{I_y} + \frac{(GI_t(KL))^2}{\pi^2 EI_y} + (C_2 y_g - C_3 y_t)^2 \right]^{0.5} - (C_2 y_g - C_3 y_t) \right\}$$

Where ,

C_1, C_2, C_3 = factors depending upon the loading and end restraint conditions

K, K_w = effective length factors of the unsupported length accounting for boundary conditions at the end lateral supports,

Here, both K and K_w can be taken as 1.0 and

$y_g = y$ distance between the point of application of the load and the shear centre of the cross-section and is positive when the load is acting towards the shear centre from the point of application

$$y_j = y_s - 0.5 \int A(z^2 - y^2)y \, dA/I_z$$

y_s = coordinate of the shear centre with respect to centroid,
positive when the shear centre is on the compression side of the
centroid.

Here, for plane and equal flange I section,

$$y_g = 0.5 \times h = 0.5 \times 0.45 = 0.225 \text{ M} = 225 \text{ mm.}$$

$$y_j = 1.0(2\beta_f - 1)h_y/2.0 \quad (\text{when } \beta_f \leq 0.5)$$

h_y = distance between shear centre of the two flanges of the
cross-section) = $h - t_f$

$$\text{Here, } \beta_f = 0.5 \text{ and } h_y = h - t_f = 450 - 17.4 = 432.6 \text{ mm}$$

$$\text{Hence, } y_j = 1.0 \times (2.0 \times 0.5 - 1)432.6/2.0 = 0 \text{ and } y_s = 0$$

$$I_t = \sum b_i t_i^3, \text{ for open section}$$

$$= 2 \times 150 \times 17.4^3 + (450 - 2 \times 17.4) \times 9.4^3$$

The warping constant, I_w is given by,

$$I_w = (1 - \beta_f) \beta_f I_y h_y^2 \text{ for I sections mono-symmetric about weak axis,}$$

$$= (1 - 0.5) \times 0.5 \times 834 \times 10^4 \times 432.6^2 = 39019265.46 \times 10^4 \text{ mm}^6$$

$$\text{Modulus of rigidity, } G = 0.769 \times 10^5 \text{ N/mm}^2$$

Here, $\psi = 86/155 = 0.555$ and $K = 1.0$ for which,

$$C_1 = 1.283, C_2 = 0 \text{ and } C_3 = 0.993$$

Hence, critical moment

$$M_{cr} = C_1 \frac{\pi^2 EI}{(KL)^2} \left\{ \left[\left(\frac{K}{K_w} \right)^2 \frac{I_w}{I_y} + \left(\frac{GI_t (KL)^2}{\pi^2 EI_y} + (C_2 Y_g - C_3 Y_1)^2 \right)^{0.5} - (C_2 Y_g - C_3 Y_t) \right] \right\}$$

$$= 1.283 \frac{\pi^2 \times 200000 \times 834 \times 10^4}{(1.0 \times 6000)^2} \left\{ \left[\left(\frac{1}{1} \right)^2 \frac{39019265 \times 10^4}{834 \times 10^4} + \frac{0.769 \times 10^5 \times 192.527 \times 10^4 \times 6000^2}{\pi^2 \times 200000 \times 834 \times 10^4} \right]^{0.5} \right\}$$

$$= 357142.72 \times 10^3 \text{ N-mm.}$$

Calculation of f_{bd} :

$$\text{Now } \lambda_{LT} = \sqrt{\beta_b z_p f_y / M_{cr}} = \sqrt{1.0 \times 1533.36 \times 10^3 \times 250 / 357142.72 \times 10^3} \\ = 1.036 \quad (\text{clause 8.2.2 of I.S 800})$$

$$\text{For which, } \Phi_{LT} = 0.5 \times [1 + \alpha_{LT}(\lambda_{LT} - 0.2) + \lambda_{LT}^2]$$

$$= 0.5 \times [1 + 0.21(1.036 - 0.2) + 1.036^2] = 1.124$$

$$\text{For which, } \chi_{LT} = \frac{1}{\{\Phi_{LT}[\Phi_{LT}^2 - \lambda_{LT}^2]^{0.5}\}}$$

$$= \frac{1}{\{1.124[1.124^2 - 1.036^2]^{0.5}\}}$$

$$f_{bd} = \chi_{LT} f_y / \gamma_{mo} = 0.641 \times 250 / 1.10 = 145.68 \text{ N/mm}^2$$

$$\begin{aligned}\text{Hence, } M_d &= \beta_b z_p f_{bd} = 1.0 \times 1533.36 \times 145.68 / 1000 \\ &= 223379.88 / 1000 \sim 223.38 \text{ kN-m.}\end{aligned}$$

Max. Bending moment $M_{\max} = 202 \text{ kN-m}$

Hence, $M_d > M_{\max} = (223.38 > 202)$

Therefore, ISMB 450 is adequate against lateral torsional buckling for the applied bending moments.

(ii) If the ISMB 450 is subjected to a central load producing a maximum factored moment of 202 kN.m , check whether the beam is still safe

For this problem with zero bending moments at the supports and central max bending moment being 202 kN-m .

For the value of $K = 1.0, C_1 = 1.365; C_2 = 0.553$ and $C_3 = 1.780$

$$M_{cr} = C_1 \frac{\pi^2 EI}{(KL)^2} \left\{ \left[\left(\frac{K}{K_w} \right)^2 \frac{I_w}{I_y} + \left(\frac{GI_t(KL)^2}{\pi^2 EI_y} + (C_2 \gamma_g - C_3 \gamma_1)^2 \right)^{0.5} - (C_2 \gamma_g - C_3 \gamma_t) \right] \right\}$$

$$= 1.365 \frac{\pi^2 \times 2 \times 10^4 \times 834 \times 10^4}{(1.0 \times 6000)^2} \left\{ \left[\left(\frac{1}{1} \right)^2 \frac{39019 \times 10^9}{834 \times 10^4} + \frac{0.769 \times 10^5 \times 192.527 \times 10^4 \times 6000^2}{\pi^2 \times 2 \times 10^5 \times 834 \times 10^4} \right]^{0.5} - 0.553 \times 225 \right\}$$

$$= 310158.31 \times 10^3 \text{ N-mm}$$

Calculation of f_{bd} :

$$\text{Now, } \lambda_{LT} = \sqrt{\beta_b z_p f_y / M_{cr}} = \sqrt{1.0 \times 1533.36 \times 10^3 \times 250 / 310158.31 \times 10^3}$$

$$= 1.112 \quad (\text{clause 8.2.2 of I.S 800})$$

$$\text{For which, } \Phi_{LT} = 0.5 \times [1 + \alpha_{LT}(\lambda_{LT} - 0.2) + \lambda_{LT}^2]$$

$$= 0.5 \times [1 + 0.21(1.112 - 0.2) + 1.112^2] = 1.214$$

$$\text{For which } \chi_{LT} = \frac{1}{\{\Phi_{LT}[\Phi_{LT}^2 - \lambda_{LT}^2]^{0.5}\}}$$

$$= \frac{1}{\{1.214[1.214^2 - 1.112^2]^{0.5}\}}$$

$$f_{bd} = \chi_{LT} f_y / \gamma_{mo} = 0.588 \times 250 / 1.10 = 133.64 \text{ N/mm}^2$$

$$\text{Hence, } M_d = \beta_b z_p f_{bd} = 1.0 \times 1533.36 \times 133.64 / 1000$$

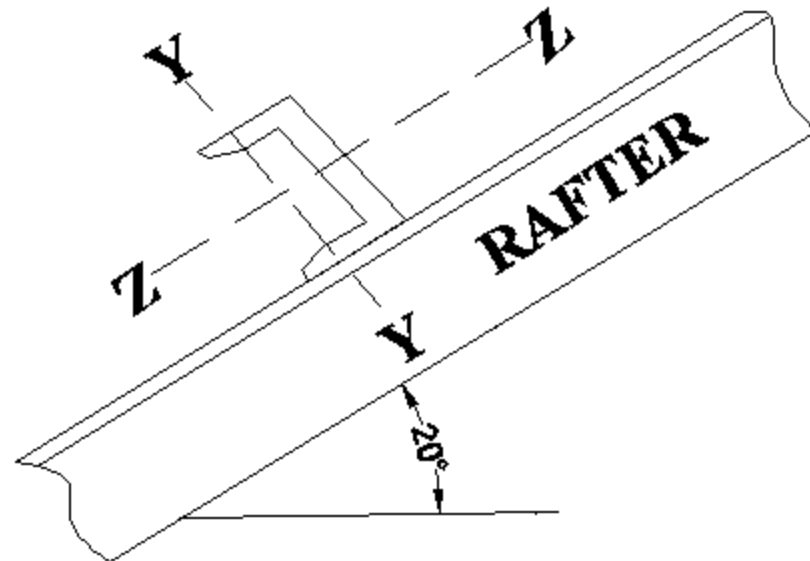
$$= 204918.23 / 1000 \sim 204.92 \text{ kN-m.}$$

4. Design a purlin on a sloping roof truss with the dead load of 0.15 kN/m^2 (cladding and insulation), a live load of 2 kN/m^2 and wind load of 0.5 kN/m^2 (suction). The purlins are 2 m centre to centre and of span **4 m, simply supported on** a rafter at a slope of 20 degrees (see Fig).

(a) Provide channel section purlin

(b) Provide channel purlin with a sag rod at mid span

(c) Provide angle purlin



Solution:

Load calculation

$$\text{Dead load} = 0.15 \times 2 = 0.3 \text{ kN/m}$$

$$\text{Live load} = 2 \times 2 = 4 \text{ kN/m}$$

$$\text{Wind load} = 0.5 \times 2 = 1 \text{ kN/m (suction)}$$

$$w_{d,} = 0.3 \times \cos 20^\circ = 0.282 \text{ kN/m}$$

$$W_{i,} = 4 \times \cos 20^\circ = 3.76 \text{ kN/m}$$

$$W_{,,} = -1 \text{ kN/m}$$

$$W_{iy} = 4 \times \sin 20^\circ = 1.37 \text{ kN/m}$$

$$w_{dy} = 0.3 \times \sin 20^\circ = 0.103 \text{ kN/m}$$

Note that W_{wy} is zero as wind pressure is perpendicular to the surface on which it acts, i.e., normal to the rafter.

Factored load combination:

Z-direction:

$$\text{WL} + \text{DL} + \text{LL} = (1.2 \times 1.0) + (1.2 \times 0.282) + (1.2 \times 3.76) = 6.0552 \text{ kN/m}$$

$$DL + LL = (1.5 \times 0.282) + (1.5 \times 3.76) = 6.063 \text{ kN/m}$$

Y-direction:

$$DL + LL = (1.5 \times 0.103) + (1.5 \times 1.37) = 2.21 \text{ kN/m}$$

Bending moment and shear force calculation:

$$M_z = 6.063 \times 4^2/8 = 12.126 \text{ kN m}$$

$$M_y = 2.21 \times 4^2/8 = 4.42 \text{ kN m}$$

$$F_z = 6.063 \times 4/2 = 12.126 \text{ kN}$$

$$F_y = 2.21 \times 4/2 = 4.42 \text{ kN}$$

(a) Channel section purlin

Assume an ISMC 200 channel.

Plastic section modulus required

$$= \frac{M_z \times \gamma_{mo}}{f_y} + 2.5 \times \frac{d}{b} \times \frac{M_y \times \gamma_{mo}}{f_y}$$

$$= \frac{12.126 \times 10^6 \times 1.10}{250} + 2.5 \times \frac{200}{75} \times \frac{4.42 \times 10^6 \times 1.10}{250}$$

$$= 183 \times 10^3 \text{ mm}^3$$

Choose a channel section ISMC 200 @ 0.22 kN/m with plastic section modulus of

$$Z_{pz} = 211.25 \times 10^3 \text{ mm}^3 \text{ and } Z_{py} = 40.716 \times 10^3 \text{ mm}^3.$$

Section Properties:

Cross sectional area $A = 2821 \text{ mm}^2$

Depth of the section $h = 200 \text{ mm}$

Width of flange $b = 75 \text{ mm}$

Thickness of flange $t_f = 11.4 \text{ mm}$

Thickness of web $t_w = 6.1 \text{ mm}$

Depth of web $d = h - 2(9 + R) = 200 - 2(11.4 + 11) = 155.2 \text{ mm}$

Elastic section modulus $Z_{ez} = 181.7 \times 10^3 \text{ mm}^3$

Elastic section modulus $Z_{ey} = 26.3 \times 10^3 \text{ mm}^3$

Plastic section modulus $Z_{pz} = 211.25 \times 10^3 \text{ mm}^3$

Plastic section modulus $Z_{py} = 40.716 \times 10^3 \text{ mm}^3$

Moment of inertia $I_{zz} = 1830 \times 10^4 \text{ mm}^4$

Moment of inertia $I_y = 141 \times 10^4 \text{ mm}^4$

Section classification:

$$\frac{t}{b_f} = \frac{75}{11.4} = 6.58 < 9.4$$

$$\frac{d}{t_w} = \frac{155.2}{6.1} = 25.44 < 42$$

Hence the section is plastic.

Calculation of shear capacity of the section Z-direction

$$V_d = \frac{f_y}{\gamma_{m0} \times \sqrt{3}} \times h \times t_w = \frac{250}{1.1 \times \sqrt{3}} \times 200 \times 6.1 = 160.18 \text{ kN}$$

$$0.6V_d = 96 \text{ kN} > 12.126 \text{ kN}$$

Y-direction

$$\text{Shear capacity} = \frac{250}{11.1 \times \sqrt{3}} \times 2 \times 75 \times \frac{11.4}{10^3} = 224.4 \text{ kN} > 4.42 \text{ kN}.$$

Note that in purlin design, the shear capacity is usually high relative to the shear force.

Design capacity of the section

$$\begin{aligned} M_{dz} &= \frac{z_{pz} \times f_y}{\gamma_{mo}} = \frac{211.25 \times 10^3}{1.1 \times 10^6} = 48 \text{ kN.m} \\ &\leq \frac{z_{pz} \times f_y}{\gamma_{mo}} = \frac{1.8 \times 181.7 \times 10^3 \times 250}{1.1 \times 10^6} = 49.55 \text{ kN.m} \end{aligned}$$

Hence, $M_{dz} = 48 \text{ kN.m} > 12.126 \text{ kN.m}$

$$\begin{aligned} M_{dy} &= \frac{z_{py} \times f_y}{\gamma_{mo}} = \frac{40.716 \times 10^3 \times 250}{1.1 \times 10^6} = 9.25 \text{ kN.m} \\ &\leq \frac{r_f \times z_{ey} \times f_y}{\gamma_{mo}} = \frac{1.5 \times 26.3 \times 10^3 \times 250}{1.1 \times 10^6} = 8.96 \text{ kN.m} \end{aligned}$$

Since the ratio z_p / z_e is greater than 1.2, the constant in the

preceding equation is replaced by the ratio of $\gamma_f = 1.5$, Hence

$$M_{dy} = 8.96 \text{ kN.m} > 4.42 \text{ kN.m}$$

Overall member strength (local capacity)

To ascertain the overall member strength, the following interaction equation should be satisfied.

$$\frac{M_z}{M_{dz}} + \frac{M_y}{M_{dy}} \leq 1$$

$$\frac{12.126}{48} + \frac{4.42}{8.96} = 0.75 \leq 1$$

Hence, the overall member strength is satisfactory

Check for deflection

$$\delta = \frac{5wl^4}{384EI} = \frac{5 \times 3.76 \times 4000^4}{384 \times 2 \times 10^5 \times 1830 \times 10^4}$$

$$\text{Allowable deflection} = \frac{l}{180} = \frac{4000}{180} = 22.22\text{mm}$$

(Table 6 of I.S 800)

Hence, the section is safe.

Check for wind suction:

The effect of wind suction has not been considered till now; it can become critical in some situations. It has to be combined with dead load

$$\text{Factored wind load } W_z = 0.9 \times 0.282 - 1.5 \times 1 = -1.246\text{kN/m}$$

$$W_y = 0.9 \times 0.103 = -0.0927\text{kN/m}$$

Buckling resistance of section

Equivalent length $l_e = 4$ m

Moment = $M_z = w l^2 / 8 = -1.246 \times 4^2 / 8 = -2.492$ kN m

$M_y = 0.0927 \times 42 / 8 = 0.1854$ kN m

The value of M_z is much lower than the value 12.126 kN m earlier, but the negative sign indicates that the lower flange of the channel is in compression and this flange is unrestrained. Hence the buckling resistance of the channel must be found.

$$M_{cr} = \sqrt{\frac{\pi^2 E I_y}{(KL)^2} \left(G I_t + \frac{\pi^2 E I_w}{(KL)^2} \right)}$$

$$G = \frac{E}{2(1 + \mu)} = \frac{2 \times 10^5}{2(1 + 0.3)} = 76.923 \times 10^3 \text{ N/mm}^2$$

$$I_t = \sum \frac{b_i t_i^3}{3} = \left[\frac{2 \times 75 \times 11.4^3}{3} + \frac{(200 - 11.4) \times 6.1^3}{3} \right] = 88346.77 \text{ mm}^4$$

$$I_w = (1 - \beta_f) \beta_f I_y h_f^2$$

$$h_f = 200 - 11.4 = 188.6 \text{ mm}$$

$$\beta_f = \frac{I_{fc}}{I_{fc} + I_{ft}} = 0.5$$

$$\begin{aligned} I_w &= (1 - 0.5) \times 0.5 \times 141 \times 10^4 \times 188.6^2 \\ &= 1.2538 \times 10^{10} \text{ mm}^6 \end{aligned}$$

$$\begin{aligned} M_{cr} &= \sqrt{\left[\frac{\pi^2 \times 2 \times 10^5 \times 141 \times 10^4}{4000^2} (76.923 \times 10^4 \times 88346.7 + \frac{\pi^2 \times 2 \times 10^5 \times 1.2538 \times 10^{10}}{4000^2}) \right]} \\ &= 38.09 \text{ kN m} \end{aligned}$$

$$\lambda_{LT} = \sqrt{\frac{\beta_b z_p f_y}{M_{cr}}}$$

$$= \sqrt{\frac{1.0 \times 211.25 \times 10^3 \times 250}{38.09 \times 10^6}} = 1.1775$$

$$\Phi_{LT} = 0.5[1 + \alpha_{LT} (\lambda_{LT} - 0.2) + \lambda_{LT}^2]$$

$$= 0.5[1 + 0.21(1.1775 - 0.2) + 1.1775^2]$$

$$= 1.296$$

$$\chi_{LT} = \sqrt{\frac{1.0}{\Phi_{LT} + [\Phi_{LT}^2 - \lambda_{LT}^2]^{0.5}}} \leq 1.0$$

$$= \sqrt{\frac{1.0}{1.296 + [1.296 - \lambda_{LT}^2]^{0.5}}} \leq 1.0$$

$$f_{bd} = \frac{\chi_{LT} f_y}{\gamma_{m0}} = \frac{0.544 \times 250}{1.10} = 123.71 \text{ N/mm}^2$$

$$\begin{aligned} M_{dz} &= z_p f_{bd} \\ &= 211.25 \times 10^3 \times 123.71 \\ &= 26.13 \text{ kNm} > 2.492 \text{ kNm} \end{aligned}$$

The buckling resistance M_{dy} of the section need not be found out, because the purlin is restrained by the cladding in the z-plane and hence instability is not considered for a moment about the minor axis .

Overall member strength

To ascertain the overall member buckling strength, the following interaction should be satisfied .

$$\frac{M_z}{M_{dz}} + \frac{M_y}{M_{dy}} \leq 1$$
$$\frac{2.492}{26.13} + \frac{0.1854}{8.96} = 0.097 < 1$$

Hence the overall member strength is satisfactory.

- It has to be noted that the maximum buckling moment occurs at the centre of the beam and the maximum shear force at the supports.
- Hence it is not necessary to check the moment capacity in the presence of shear force.
- Also purlins are not normally checked for web bearing and crippling

as the applied concentrated loads are low (note the low value of Shear force)

(b) Channel section purlin with one sag rod at mid span

Since the channel section purlin is provided with a sag rod at mid – span, the bending moment in the y- direction will be reduced considerably .

$$M_y = 2.21 \times 4^2 / 32 = 1.105 \text{ kN m}$$

$$M_z = 12.126 \text{ kN m}$$

$$\text{Required section modulus} = (M_z \times \gamma_{m0} / f_y) + 2.5(d/b)(M_y \times \gamma_{m0} / f_y)$$

Assuming ISMC 100 with $d = 100 \text{ mm}$ and $b = 50 \text{ mm}$,

$$\begin{aligned} \text{Required } Z &= (12.126 \times 10^6 \times 1.1 / 250) \\ &= 77.66 \times 10^3 \text{ mm}^3 \end{aligned}$$

Provide ISMC 150 with following section properties

Depth of section $h = 150\text{mm}$; $r_y = 22\text{ mm}$

Width of flange $b = 75\text{ mm}$

Thickness of flange $t_f = 9.0\text{ mm}$

Thickness of web $t_w = 5.7\text{ mm}$

Elastic section modulus $z_{ez} = 105 \times 10^3 \text{mm}^3$

Elastic section modulus $z_{ey} = 19.5 \times 10^3 \text{mm}^3$

Plastic section modulus

$$z_{pz} = 119.5 \times 10^3 \text{mm}^3 > 77.66 \times 10^3 \text{mm}^3$$

Moment of inertia $I_{pz} = 788 \times 10^4 \text{mm}^3$

Section classification

$$b/t_f = 75/9.0 = 8.33 < 9.4$$

$$d/t_w = [150 - 2(9.0 + 10)] / 5.7 = 19.65 < 42$$

Hence the section is plastic. Shear capacity is not being checked since the shear force is small and hence the section will be adequate.

Design capacity of the section

$$\begin{aligned} M_{dz} &= (z_{pz} \times f_y / \gamma_{m0}) \\ &= (119.83 \times 10^3 \times 250 / 1.1 \times 10^6) = 27.23 \text{ kN m} \\ &\leq (1.2 \times z_{ez} f_y / \gamma_{m0}) = [(1.2 \times 105 \times 10^3 \times 250) / (1.1 \times 10^6)] \\ &= 28.63 \end{aligned}$$

$$\begin{aligned} Z_{py} &= 2t_f b_f^2 / 4 + (h - 2t_f) t_w^2 / 4 = 2 \times 9.0 \times 75^2 / 4 + (150 - 2 \times 9.0) \\ &5.7^2 / 4 = 26384.6 \text{ mm}^3 \end{aligned}$$

$$\begin{aligned}
M_{dy} &= (z_{py} f_y / \gamma_{m0}) \\
&= (26384.6 \times 250 / 1.1 \times 10^6) = 6.0 \text{ kN m} \\
&\leq (1.5 \times z_{ey} f_y / \gamma_{m0}) = 1.5 \times (19.5 \times 10^3 \times 250) / (1.1 \times 10^6) \\
&= 6.6 \text{ kN m}
\end{aligned}$$

Hence the section is safe.

Overall member strength

For overall member strength, the following interaction equation must be satisfied.

$$\begin{aligned}
(M_z / M_{dz}) + (M_y / M_{dy}) &\leq 1.0 \\
(12.126 / 27.23) + (1.105 / 6.0) &= 0.629 < 1.0
\end{aligned}$$

Hence the member strength is satisfactory.

Check for deflection

$$\begin{aligned}\delta &= (5wl^4/384EI) = (5 \times 3.76 \times 4000^4) / (384 \times 2 \times 10^5 \times \\ &\quad 788 \times 10^4) \\ &= 7.95 \text{ mm} < 22.22 \text{ mm}\end{aligned}$$

Hence the section is safe.

Check for wind suction

From part (a) , $M_z = 2.492 \text{ kN m}$

$$M_y = 0.0927 \times 4^2/32 = 0.0464 \text{ kN m}$$

$$f_{cr} = [1473.5 / (KL/r_y) / (h/t_f)]^2 \}^{0.5}$$

$$KL/r_y = 4000/22 = 181.8$$

$$h/t_f = 150/9.0 = 16.67$$

Thus, $f_{cr} = (1473.5/11.8)^2 \{1 + (1/20) [181.8/16.67]^2\}^{0.5}$

$$=173.1 \text{ N/mm}^2$$

$$f_{bd} = 120.0 \text{ N/mm}^2 \text{ (from table 13a of the code)}$$

$$M_{dz} = Z_{pz} f_{bd} = 119.82 \times 10^3 \times 120.0/10^6 = 14.38 \text{ kN m}$$

Overall buckling strength

For overall buckling strength, the following interaction equation should be satisfied.

$$\begin{aligned} (M_z / M_{dz}) + (M_y / M_{dy}) &= (2.492/14.38) + (0.0464/6.0) \\ &= 0.18 < 1.0 \end{aligned}$$

Hence the overall buckling strength is satisfactory.

Hence by using one sag rod, it was possible to reduce the section from ISMC 200 to ISMC 150 (about 25% reduction in weight).

(c) *Angle Section Purlin (as per BS 5950-1:2000)*

From part (a) $M_z = 12.126 \text{ kN m}$; $W_p = (1.0 + 0.282 + 3.76) \times 4$
 $= 20.168 \text{ kN}$

Moment at working load $= 12.126 / 1.5 = 8.084 \text{ kN m}$

Let us assume that bending about z-z axis resists the vertical loads and the horizontal component is resisted by the sheeting.

Design strength $f_y = 250 \text{ Mpa}$

Applied moment = moment capacity of single angle

$$8.084 \times 10^6 = 250 \times Z_{ez}$$

$$\text{Required } Z_{ez} = 8.084 \times 10^6 / 250 = 32.33 \times 10^3 \text{ mm}^3$$

Provide ISA 150 x 75 x 10 angle @ 0.17 kN/m,

With $Z_{ez} = 51.9 \times 10^3 \text{ mm}^3 > 20.168 \times 4 \times 10^6 / 1800 = 10^3 \text{ mm}^3$

$$=44.817 \times 10^3 \text{mm}^3$$

$$d/t = 150/10 = 15.0 > 10.5 \text{ but } < 15.7$$

The section is *semi – compact*.

Leg length perpendicular to plane of cladding
 $= 4000/45 = 88.88 \text{ mm} < 150 \text{ mm}$

Leg length parallel to plane of cladding
 $= 4000/60 = 66.66 \text{ mm} < 75 \text{ mm}$

Deflection need not be checked in this case.

Thank You

DESIGN OF STEEL STRUCTURES

by

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***COMPRESSION
MEMBERS***

TYPES OF COMPRESSION MEMBERS

The types of column is based on the slenderness ratio or the length to diameter ratio of the columns.

They can be divided as follows:

(a) Short columns:

- The columns which have height less than eight times their diameter or slenderness ratio less than 32 are called short column.
- In short columns bending or buckling is negligible and hence short columns fail by direct crushing or compressive stress.

(b) Medium columns:

- The columns which have length varying from 8 to 30 times their diameters or the slenderness ratio lying between 32 to 120 are called intermediate or medium columns.

- In this types of columns, buckling and compressive stress are both considered for their failures.

(c) Long columns :

- Columns having their length more than 30 times their diameter or slenderness ratio more than 120 are called long columns.
- In such types of columns failure will occur due to buckling or bending but direct compressive stress is very small as compared to buckling stress.

SLENDERNESS RATIO

- The slenderness ratio of a member is the ratio of the effective length to the appropriate radius of gyration (KL/r).
- This valid only when the column has equal unbraced height for both axes and end condition are same for both axes. The appropriate radius of gyration is one which is minimum for a particular section.
- For example a section asymmetrical about the centroidal axes will bend about the principal axis for which the radius of gyration is minimum.
- On the other hand, a section symmetrical about both the centroidal axes (I-section) or even with one axis of symmetry (channel section, two angles back to back) will bend about one of the centroidal axis giving lesser radius of gyration.
- This is because for such section the principal axes coincide with the centroidal axes.

SLENDERNESS RATIO

The slenderness ratio of compression member is limited because of the following reason:

1. The effect of accidental and construction (fabrication, transportation, and erection) loads are automatically taken care of.
2. The bracing members may be used as a walkway for workmen or to provide temporary support for equipments.
3. To take care of the probability of member being subjected to unexpected vibrations.

CODAL PROVISIONS

- Design compressive stress cannot be more than f_y .
- Reduction in f_y due to all the above adverse factors is difficult to quantify.
- Based on statistical test data, lower bound curves are proposed as shown in above Figure.
- The curves a, b, c and d represent considering the effects of cumulative degree of imperfection due to cross-sectional layout, presence of residual, initial curvature and eccentric loading.

CODAL PROVISIONS

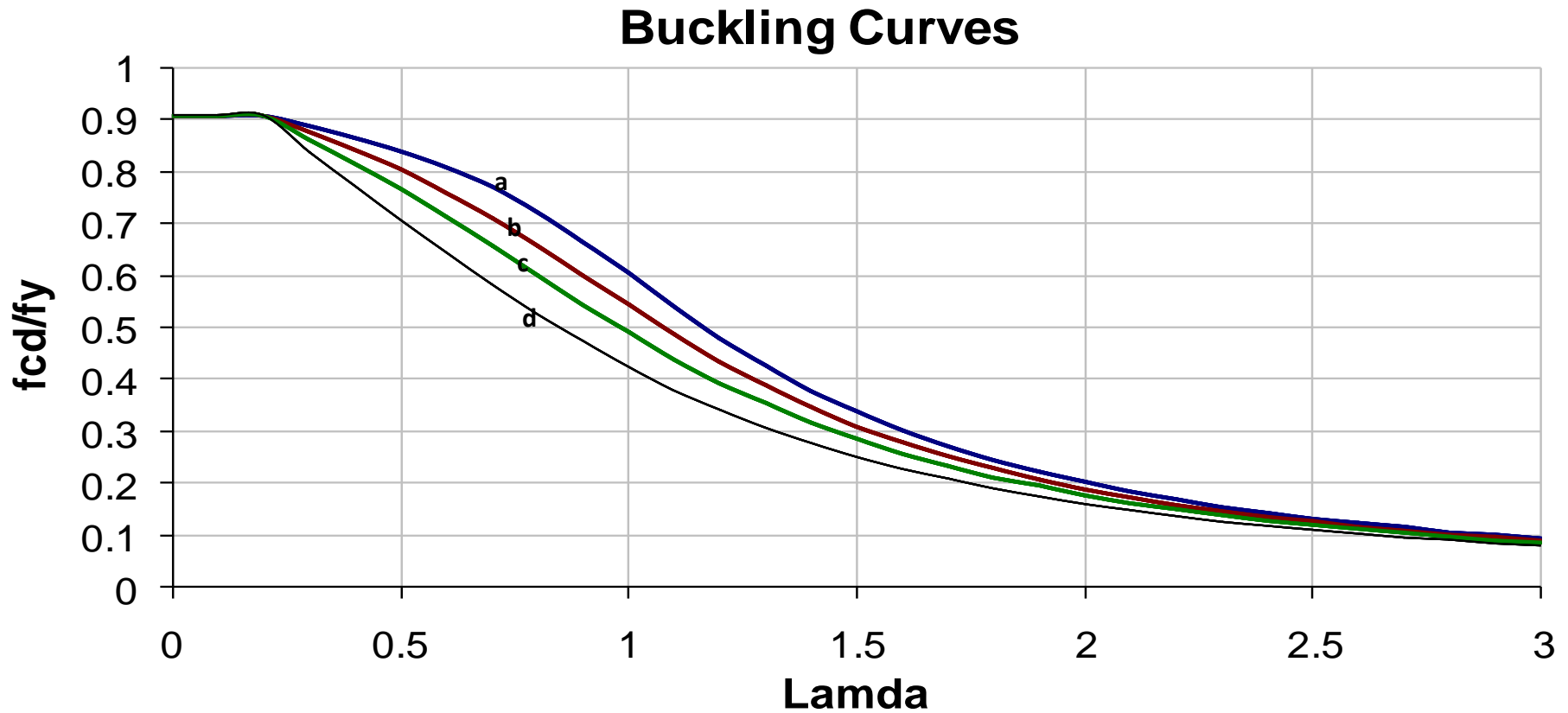


TABLE 1 IMPERFECTION FACTOR, α

Buckling Class	a	b	c	d
α	0.21	0.34	0.49	0.76

DESIGN STRENGTH

The design compressive strength of a member is given

by $P_d = A_e f_{cd}$

$$f_{cd} = \frac{f_y / \gamma_{m0}}{\phi + [\phi^2 - \lambda^2]^{0.5}} = \chi f_y / \gamma_{m0} \leq f_y / \gamma_{m0}$$
$$\phi = 0.5[1 + \alpha(\lambda - 0.2) + \lambda^2]$$

$$\lambda = \sqrt{f_y / f_{cc}} = \sqrt{f_y (KL/r)^2 / \pi^2 E}$$

f_{cd} = the design compressive stress,

λ = non-dimensional effective slenderness ratio,

f_{cc} = Euler buckling stress = $\pi^2 E / (KL/r)^2$

α = imperfection factor as in Table 1

χ = stress reduction factor as in Table 8

γ_{m0} = partial safety factor for material

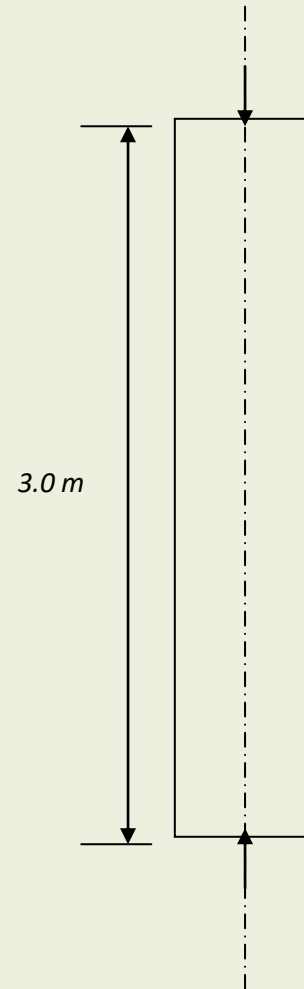
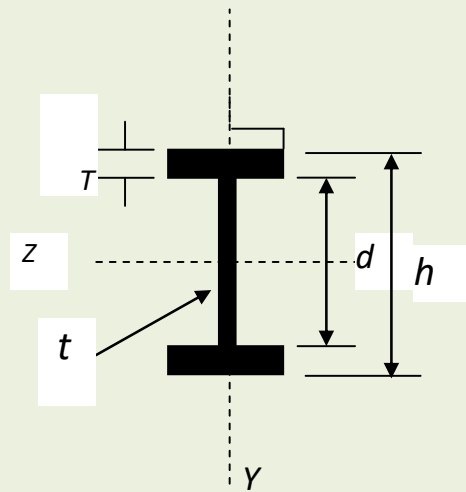
KL/r = Effective slenderness ratio

$$\chi = \frac{1}{\left[\phi + (\phi^2 - \lambda^2)^{0.5} \right]}$$

1. Obtain factored axial load on the column section ISHB400. The height of the column is 3.0m and it is pin-ended.

[$f_y = 250 \text{ N/mm}^2$; $E = 2 \times 10^5 \text{ N/mm}^2$; $\gamma_m = 1.10$]

CROSS-SECTION PROPERTIES:



Flange thickness	=	T	=	12.7 mm
Overall height of ISHB400	=	h	=	400 mm
Clear depth between flanges	=	d	=	$400 - (12.7 \times 2)$ = 374.6 mm
Thickness of web	=	t	=	10.6mm
Flange width	=	2b	=	$b_f = 250$ mm
Hence, half Flange Width	=	b	=	125 mm
Self –weight	=	w	=	0.822 kN/m
Area of cross-section	=	A	=	10466 mm ²
Radius of gyration about x	=	r_x	=	166.1 mm
Radius of gyration about y	=	r_y	=	51.6 mm

(i) Type of section:

$$\frac{b}{T} = \frac{125}{12.7} = 9.8 < 10.5\varepsilon$$

$$\frac{d}{t} = \frac{374.6}{10.6} = 35.3 < 42\varepsilon \quad (\text{Table 3.1 of IS: 800})$$

$$\text{where, } \varepsilon = \sqrt{\frac{250}{f_y}} = \sqrt{\frac{250}{250}} = 1.0$$

Hence, cross-section can be classified as “COMPACT”.

(ii) Effective Sectional Area, $A_e = 10,466 \text{ mm}^2$

(Since there is no hole, (Clause 7.3.2 of IS: 800)

no reduction has been considered)

(iii) Effective Length:

As, both ends are pin-jointed effective length

(Clause 7.2 and Table 7.5 of IS:800)

$$KL_x = KL_y = 1.0 \times L_x = 1.0 \times L_y = 1.0 \times 3.0 \text{ m} = 3.0 \text{ m}$$

(iv) Slenderness ratios:

$$KL_x / r_x = \frac{3000}{166.1} = 18.1$$

$$KL_y / r_y = \frac{3000}{51.6} = 58.1$$

(v) Non-dimensional Effective Slenderness ratio, λ :

$$\begin{aligned} \lambda &= \sqrt{f_y / f_{cc}} = \sqrt{f_y (KL/r)^2 / \pi^2 E} = \sqrt{250 \times (58.1)^2 / \pi^2 \times 2 \times 10^5} \\ &= 0.654 \quad \text{(Clause 7.1.2.1 of IS: 800)} \end{aligned}$$

(vi) *Value of ϕ from equation $\phi = 0.5[1 + \alpha(\lambda - 0.2) + \lambda^2]$:*

Where, α = Imperfection Factor which depends on Buckling Class

Now, from Table 7.2 of Chapter 7, for $h/b_f = 400 / 250 = 1.6 > 1.2$ and also thickness of flange, $T = 12.7$ mm, hence for z-z axis buckling class 'a' and for y-y axis buckling class 'b' will be followed.

(Table 7.1 of IS: 800)

Hence, $\alpha = 0.34$ for buckling class 'b' will be considered.

Hence, $\phi = 0.5 \times [1 + 0.34 \times (0.654 - 0.2) + 0.654^2] = 0.791$

(Table 7.1 of IS: 800)

(vii) **Calculation of χ from equation $\chi = \left[\frac{1}{\left[\phi + (\phi^2 - \lambda^2)^{0.5} \right]} \right]$**

$$\chi = \left[\frac{1}{\left[\phi + (\phi^2 - \lambda^2)^{0.5} \right]} \right] = \left[\frac{1}{\left[0.791 + (0.791^2 - 0.654^2)^{0.5} \right]} \right]$$

$$= 0.809$$

(vii) **Calculation of f_{cd} from the following equation:**

$$f_{cd} = \frac{f_y / \gamma_{m0}}{\phi + \left[\phi^2 - \lambda^2 \right]^{0.5}} = \chi f_y / \gamma_{m0} = 0.809 \times 250 / 1.10 = 183.86 \text{ N/mm}^2$$

(ix) **Factored axial load in kN.**

$$P_d = A_g f_{cd} = 10466 \times 183.86 / 1000 = 1924.28 \text{ kN.}$$

2. A double angle discontinuous strut ISA 150x75x10mm long leg back to back is connected to either side by gusset plate of 10mm thick with 2 bolts. The length of the strut between the intersections is 3.5m. Determine the safe load carrying capacity of the section.

Ref. CL 7.5.2.1, P48, IS800:2007

Effective length factor is between 0.7 and 0.85

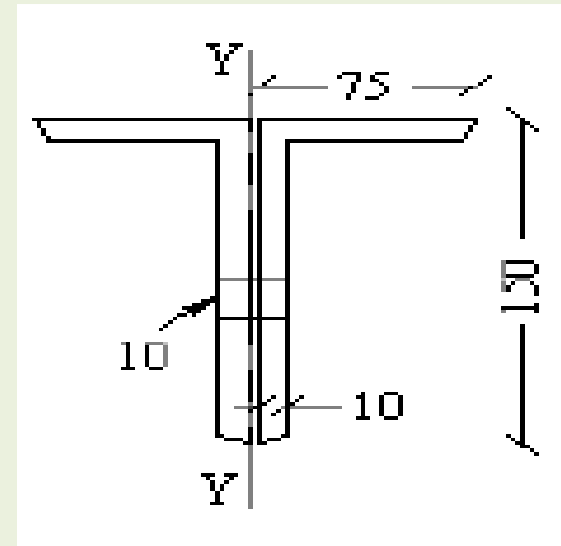
Assume $k=0.85$

Effective length of the member = 0.85×3500
= 2975mm

From steel table (P 45) $A= 4312\text{mm}^2$; $r_{\min}= 29.0$

$KL/r_{\min} = 2975/29.0 = 102.58$

From Table 10/IS 800/ P 44



The given section is belonged to **Buckling Class C**

Therefore Design Compressive stress, from **Table 9(c)/P42**

$$f_{cd} = 107 - 2.58 \times 12.4/10 = 103.8$$

$$\text{Strength of member} = (103.8 \times 4312) / 1000 = 447.58 \text{ kN}$$

3. Calculate the safe load of a bridge compression member of two channels ISMC 350 @ 421.1 kg/m placed toe to toe. The effective length of member is 7m. The widths over the back of the channel are 350mm and the section is properly connected by lacings.

$$A = 2(53.66) = 107.32 \text{ cm}^2$$

$$I_{zz} = 2(10008) = 20016 \text{ cm}^4$$

$$I_{yy} = 2[430.6 + 53.66(17.5 - 2.44)^2]$$

$$= 25201.7 \text{ cm}^4$$

$$\gamma_{\min} = \sqrt{I_{\min}/A}$$

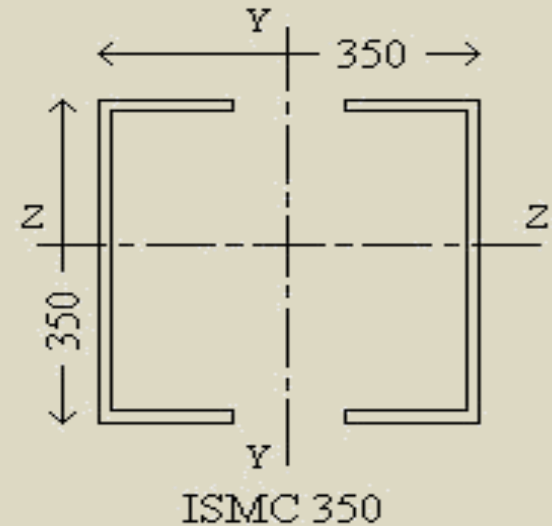
$$= 13.6 \text{ cm}$$

$$KL/\gamma = 700/13.6 = 51.2$$

(Table 9c of code)

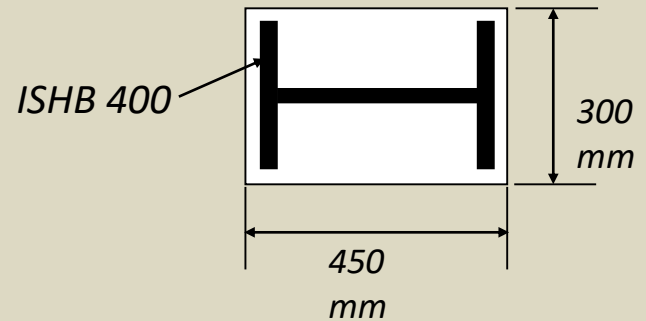
$$f_{cd} = 183 - 1.2/10 \times 15 = 181.2 \text{ N/mm}^2$$

$$\text{Strength of the member} = 181.2 \times 10732 = 1944.6 \text{ kN}$$



4. Design a simple base plate for a ISHB400 @ 0.822 kN/m column to carry a factored load of 1800 kN.

$$[f_{cu} = 40 \text{ N/mm}^2; f_y = 250 \text{ N/mm}^2; \gamma_m = 1.10]$$



Thickness of Flange for ISHB400 = T = 12.7 mm

Bearing strength of concrete = $0.4f_{cu} = 0.4 \times 40 = 16 \text{ N/mm}^2$

Area required = $1800 \times 10^3 / 16 = 112500 \text{ mm}^2$

Use plate of 450 X 300 mm (135000 mm^2)

Assuming projection of 25 mm on each side, a = b = 25 mm

$$w = (1800 \times 10^3 / 450 \times 300) = 13.33 \text{ N/mm}^2$$

Now thickness of Slab Base, t_s

Clause 7.4.3.1 of IS: 800

$$t_s = \sqrt{2.5 w (a^2 - 0.3b^2) \gamma_{m0} / f_y} > T$$

$$= \sqrt{\frac{2.5 w (a^2 - 0.3b^2) \times 1.10}{f_y}} = \sqrt{\frac{2.5 \times 13.33 \times (25^2 - 0.3 \times 25^2) \times 1.10}{250}} = 8.01 \text{ mm}$$

< T = 12.7 mm, Hence provide a base plate of thickness not less than 12.7 mm and since the available next higher thickness of plate is 16 mm

Use 450 X 300 X 16 mm plate.

5. Design a laced column 10-m long to carry a factored axial load of 1100 kN.

The column is restrained in position but not in direction at both ends. Provide single lacing system with bolted connection.

(a) Design the column with two channels back-to-back

(b) Design the column with two channels placed toe-to-toe

(c) Design the lacing system with welded connections for channels back-to-back.

Solution:

Design of column:

$$P = 1100 \times 10^3 \text{ N}$$

$$L = 1.0 \times 10 = 10\text{m}$$

Assume design strength of 125MPa

$$\text{Required area} = 1100 \times 10^3 / 125 = 8800\text{mm}^2$$

Select two ISMC 300 at 363 N/m. The relevant properties of ISMC 300 are

$$A = 4630 \text{ mm}^2, r_{zz} = 118.0 \text{ mm}, r_{yy} = 26.0 \text{ mm}$$

$$C_{yy} = 23.5 \text{ mm}, I_{zz} = 6420 \times 10^4 \text{ mm}^4, I_{yy} = 313 \times 10^4 \text{ mm}^4$$

$$\text{Area available} = 2 \times 4630 = 9260 \text{ mm}^2$$

Built up sections will be economical, when the radius of gyration of the y-y axis is increased in such a way that it is more or less equal to the radius of Gyration about the z-z axis .This is achieved by spacing the sections in such a way that r_{zz} becomes r_{\min} .Let us first check the safety of the section and then Workout the required spacing between the two channels.

$$L/r_{zz} = 10 \times 10^3 / 118.0 = 84.74$$

$$\begin{aligned} \text{The } L/r \text{ of the built-up column should be taken as } 1.05 \times (L/r_{zz}) &= 1.05 \times 84.74 \\ &= 88.98 \end{aligned}$$

For $L/r_{zz} = 88.98$ and $f_y = 250 \text{ MPa}$, using table 9c of the code ,

$$f_{cd} = 122.53 \text{ MPa},$$

Load carrying capacity $A_e f_{cd} = 9260 \times 122.53/1000$
 $= 1135 \text{ kN} > 1100 \text{ kN}.$

Hence the column is safe.

(a) Let us provide two channels back to back and connect them by lacing and denote S as the spacing between two channels[See fig below].

Spacing of channels:

$$2I_{zz} = 2 [I_{yy} + A(S/2) + c_{yy})^2]$$

$$\text{Thus, } 2 \times 6420 \times 10^4 = 2 \times [313 \times 10^4 + 4630$$

$$(S/2 + 23.5)^2]$$

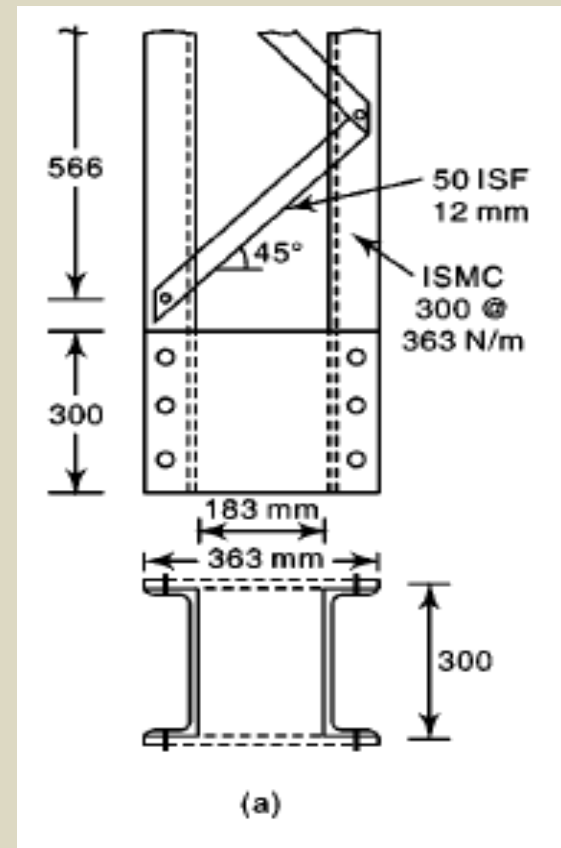
$$= 13190$$

$$S = 182.70 \text{ mm}$$

Let us keep the channels at a spacing of 183mm.

Lacing system

Using single lacing system with the inclination of



lacing bar = 45° (gauge length for a 90 mm flange = 50mm)

$$\begin{aligned} \text{Spacing of lacing bars, } L_o &= (2 \times 183 + 50 + 50) \cot 50^\circ \\ &= 2 \times 283 \times 1 = 566 \text{ mm} \end{aligned}$$

L_o / r_{yy} should be $< 0.7 \times L/r$ of whole column.

$$21.77 < 0.7 \times 88.98 = 62.3$$

Hence safe.

$$\text{Maximum shear} = (2.5 / 100) \times 1100 \times 10^3 = 27,500 \text{ N.}$$

$$\text{Transverse shear in each panel} = (V/N) = 27500/2 = 13750 \text{ N.}$$

$$\begin{aligned} \text{Compressive force in the lacing bar} &= (V/N) \operatorname{cosec} 45^\circ \\ &= 13750 \times 1.414 = 19445 \text{ N.} \end{aligned}$$

Assuming 16-mm diameter bolts,

$$\text{Minimum width of lacing flat (clause 7.6.2 of the code)} = 3 \times 16, \text{ say } 50 \text{ mm}$$

$$\text{Minimum thickness} = (1/40) (183 + 50 + 50) \operatorname{cosec} 45^\circ = 10.01 \text{ mm}$$

Provide 12 mm thick plate with a width of 50 mm

Minimum $r = t / \sqrt{12} = 12 / \sqrt{12} = 3.464 \text{ mm}$

L/r of the lacing bar = $283 \times \operatorname{cosec} 45^\circ / 3.464 = 115.5 < 145$

Hence safe.

For $L/r = 115.5$ and $f_y = 250 \text{ MPa}$, using table 9c of the code

$f_{cd} = 88.6 \text{ Mpa}$

Load carrying capacity = $88.6 \times 50 \times 12 = 53,163 \text{ N} > 19,445 \text{ N}$.

Hence the lacing bar is safe.

Tensile strength of lacing flat = $0.9(B-d)t f_u / \gamma_{m1}$ or $f_y A_g / \gamma_{m0}$

Thus $0.9(50-18) \times 12 \times 410 / 1.25$ or $250 \times 50 \times 12 / 1.1$

$113,356 \text{ N}$ or $136,363 \text{ N}$.

Thus, the tensile strength of the lacing flat = $113,356 \text{ N} > 19,445 \text{ N}$

Hence, the lacing flat is safe

Check,

r_{\min} of the built-up column = 118 mm

r_{\min} of the individual chords=26.0mm

$$L_o / r = 566/26 = 21.77$$

λ of the built up column

$$\begin{aligned}\lambda_e &= \sqrt{\{84.74^2 + 3.142(9260/600) \times 400.223 / (566 \times 2302)\}} \\ &= 86.64 < 88.98\end{aligned}$$

Hence , the column is safe.

Connection: Assuming that the 16mm bolts of grade 4.6 are connecting both lacing flats with the channel at one point and that the shear plane will not pass through the threaded portion of bolt.

$$\begin{aligned}\text{Strength of bolt in double shear} &= 2 \times A_{sb} (f_u / \sqrt{3}) / \gamma_{mb} \\ &= 2 \times \pi \times 16^2 / 4 \times (400 / \sqrt{3}) / 1.25 = 74,293\text{N}\end{aligned}$$

$$\begin{aligned}\text{Strength in bearing} &= 2.5 k_b d t f_u / \gamma_{mb} \\ &= 2.5 \times 0.49 \times 16 \times 12 \times 410 / 1.25 \\ &= 77,145 \text{ N}\end{aligned}$$

Hence ,Strength of bolt =74,293N >19,445N

Hence one 16-mm diameter bolt of grade 4.6 is required.

Connection Assuming that the 16mm bolts of grade 4.6 are connecting both lacing flats with the channel at one point and that the shear plane will not pass through the threaded portion of bolt.

$$\begin{aligned}\text{Strength of bolt in double shear} &= 2 \times A_{sb}(f_u/\sqrt{3})/\gamma_{mb} \\ &= 2 \times \pi \times 16^2/4 \times (400/\sqrt{3})/1.25 = 74,293\text{N}\end{aligned}$$

$$\begin{aligned}\text{Strength in bearing} &= 2.5 k_b d t f_u/\gamma_{mb} \\ &= 2.5 \times 0.49 \times 16 \times 12 \times 410/1.25 \\ &= 77,145 \text{ N}\end{aligned}$$

Tie plates:

Tie plates must be provided at the ends of the laced column

$$\begin{aligned}\text{Effective depth} &= 183 + 2 \times C_{yy} > 2 \times b_f \\ &= 183 + 2 \times 23.5 = 230\text{mm} > 2 \times 90 = 180\text{mm}\end{aligned}$$

Hence ,

Required overall depth of tie plate = $230 + 2 \times 25 = 280\text{mm}$ (edge distance of 16-mm diameter bolts = 25mm)

Provide a tie plate of 300 mm depth

Length of tie plate = $183 + 2 \times 90 = 363\text{mm}$

Required thickness of tie plate = $1/50 (183 + 2g) = 1/50(183 + 2 \times 50) = 5.66\text{mm}$
(where g = gauge distance –(see appendix D))

Hence ,provide a tie plate of 6-mm thickness

Provide a tie plate of size = $363 \times 300 \times 6$ mm at both ends with six 16-mm diameter bolts .

(b) Consider the case of laced columns with two channels provided toe – to – toe

Spacing:

$$2I_{zz} = 2 [I_{yy} + A (S/2 - C_{yy})^2] = 13190$$

$$S = 276.7\text{mm}$$

Let us place the channel at a spacing of 280mm

Connecting system

Assuming single lacing system is provided with an inclination of 45° ; gauge length for 90mm flange = 50mm

$$L_o = (280 - 50 - 50) \cot 45^\circ = 360 \text{ mm}$$

$$L_o / r_{yy} = 360 / 26 = 13.8 < 50$$

Hence L_o / r_{yy} ratio is fine

$$0.7(L/r) \text{ of combined channel} = 0.7 \times 88.98 = 62.3 > 13.8$$

Hence, L/r ratio is ok.

Compressive force in lacing bar = 19,445N.

Minimum width of lacing flat for 16mm bolt (clause 7.6.2 of code)
= 50mm

$$\begin{aligned} \text{Minimum thickness} &= 1/40 (280 - 50 - 50) \times \operatorname{cosec} 45^\circ \\ &= 6.36 \text{ mm} \end{aligned}$$

Hence, Provide a 50 x 8 mm flat

Check $r_{\min} = t/\sqrt{12} = 8/\sqrt{12} = 2.309\text{mm}$

$$L/r = 180 \times \operatorname{cosec} 45^\circ / 2.309 = 110.2 < 145$$

Hence, the chosen flat is safe.

For , $L/r = 110.2$ and $f_y = 250 \text{ MPa}$,from table 9c of the code

$$f_{cd} = 94.4 \text{ N/mm}^2$$

$$\begin{aligned} \text{Capacity of the lacing flat} &= 94.4 \times 50 \times 8 \\ &= 37,760 \text{ N} > 19,445 \text{ N}. \end{aligned}$$

Tensile Strength of lacing flat = $0.9(B - d)tf_u/\gamma_{m1}$ or $f_y A_g/\gamma_{m0}$

$$= 0.9 (50-18) \times 8 \times 410/1.25 \text{ or } 250 \times 50 \times 8/1.1$$

$$= 75, 571 \text{ N or } 90,909 \text{ N both} > 19,445 \text{ N}$$

Hence ,the lacing flat is safe .

Connection:

Strength of bolt in double shear {from a }=74,293N

$$\begin{aligned}\text{Strength in bearing} &= 2.5k_b d t f_u / \gamma_{mb} = 2.5 \times 0.49 \times 16 \times 8 \times 410 / 1.25 \\ &= 51,430\text{N}\end{aligned}$$

Hence, Strength of bolt = 51,430N > 19,445N.

Therefore, provide one 16-mm diameter bolts of grade 4.6

Tie plate:

$$\begin{aligned}\text{Effective depth of tie plate} &= S - 2C_{yy} \\ &= 280 - 2 \times 23.5 = 233\text{mm} > 2 \times 90 = 180\text{mm}\end{aligned}$$

Required overall depth = 230 + 2 x 25 = 280mm (edge distance of 16 mm diameter bolt = 25mm)

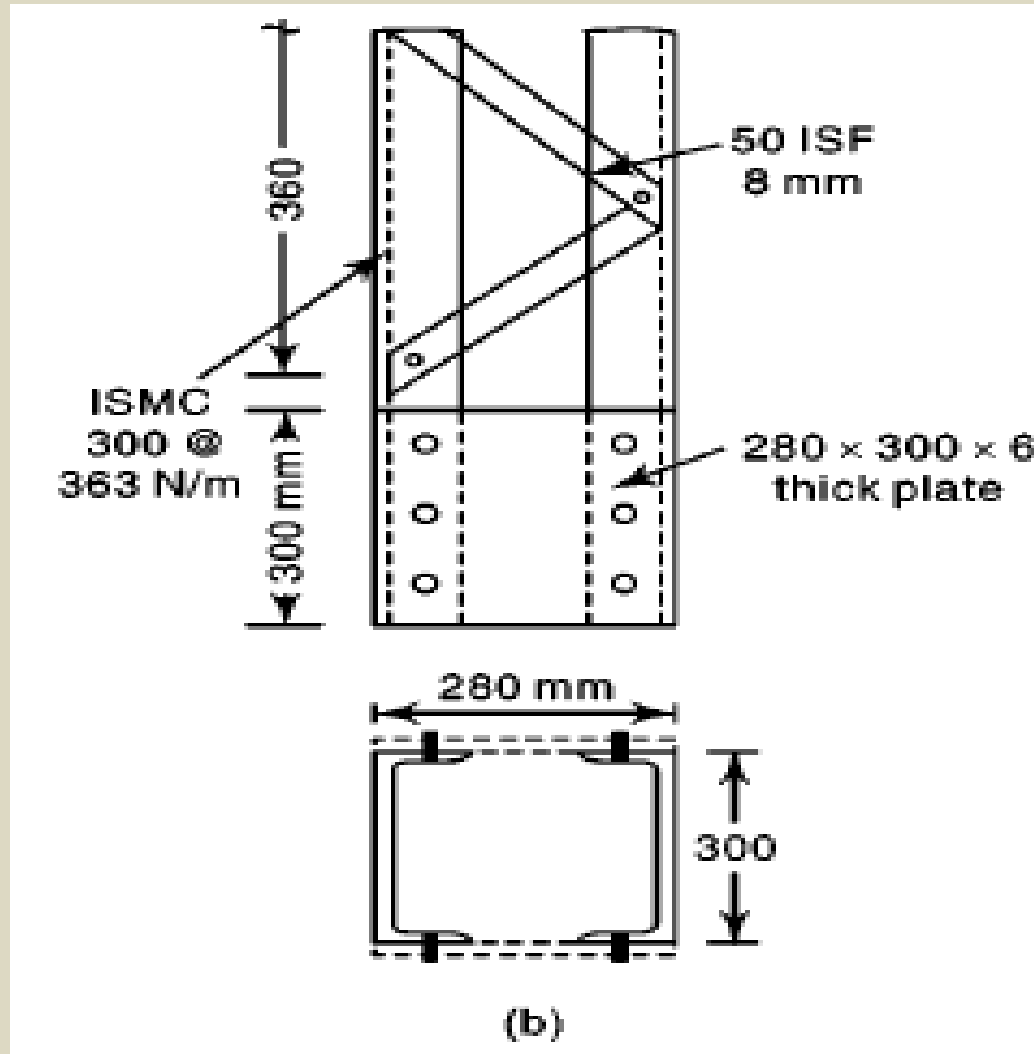
Provide a 300mm plate .

Length of tie plate = 280mm

Thickness of tie plate = (1/50)(280 - 2 X 50) = 3.6mm

Provide 6 mm

Provide a tie plate of size 280 x 300x 6 mm and use six of bolts 16-mm diameter and grade 4.6 to connect it to the channels. The arrangement is shown in fig (b)



It is seen that by providing channels toe-to-toe ,the lacing size and the tie plate Size are reduced.

(c) From part (a)

Spacing of the channels =183mm

Compressive force in the lacing = 19,445N.

Effective length of lacing flat (welded) = $0.7 \times 183 \times \operatorname{cosec} 45^\circ = 181.16\text{mm}$

Minimum thickness of flat = $\frac{1}{40} \times (183 \times \operatorname{cosec} 45^\circ)$
= 6.47 mm

Provide 50 x 8 mm lacing flat.

Minimum radius of gyration , $r = \frac{t}{\sqrt{12}} = \frac{8}{\sqrt{12}} = 2.31 \text{ mm.}$

$$L/r = 181.16 / 2.31 = 78.4 < 145.$$

Hence the L/r ratio is ok.

For $L/r = 78.4$ and $f_y = 250\text{MPa}$, Using table 9c of the code

$$f_{cd} = 138.56 \text{ N/mm}^2$$

Capacity of lacing bar = $138.56 \times 50 \times 8 = 55,424 \text{ N} > 19,445\text{N}$

Hence, the lacing bar is safe.

Overlap of lacing flat = $50\text{mm} > 4 \times 8 = 32 \text{ mm}$

Hence, the lacing flat is safe.

Connection:

Thickness of flange of ISMC 300 = 13.6mm

Minimum size of weld = 5 mm (Table 21 of code)

Strength of weld/unit length = $0.7 \times 5 \times 410 / (\sqrt{3} \times 1.5)$
 $= 552 \text{ N/mm}$

Required length of weld = $19,445 / 552 = 35.2 \text{ mm}$

Adding extra length for ends, the weld length to be provided
 $= 36 + 2(2 \times 5) = 56\text{mm}$

Provide 100mm weld length at both ends.

Tie plate:

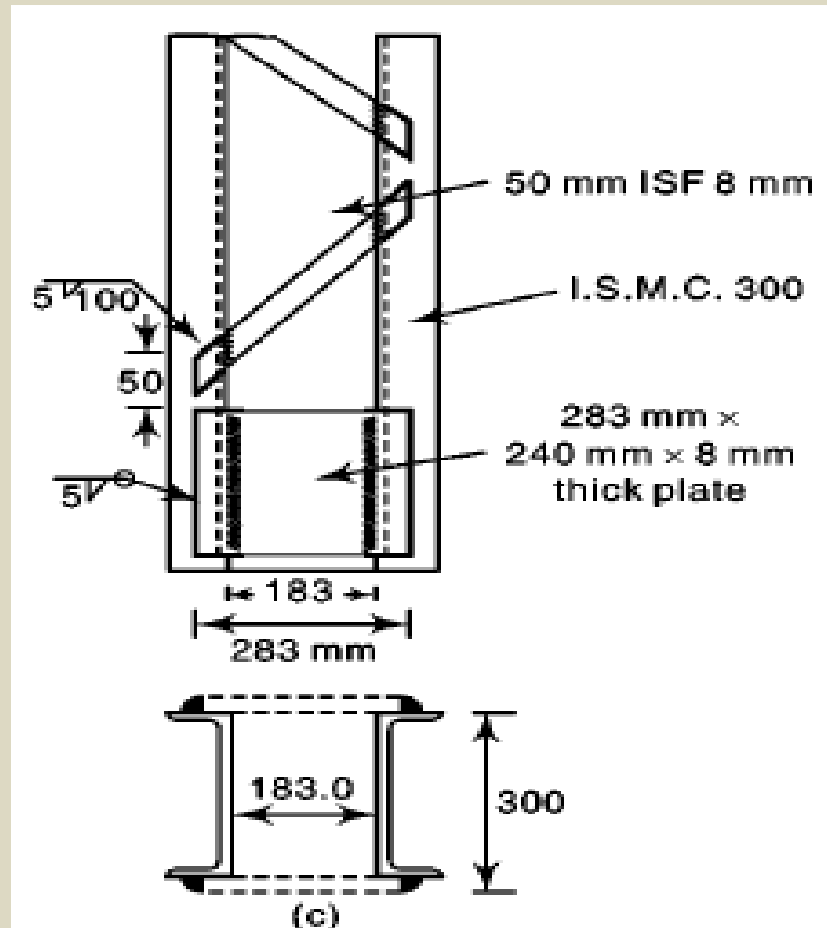
Overall depth of plate = $183 + 2 \times C_{yy}$
 $= 183 + 2 \times 23.5$
 $= 230\text{mm} > 2 \times 90\text{mm}$

Let, length of tie plate = $183 + 2 \times 50 = 283$ mm

Thickness of tie plate = $1/50(183 + 2 \times 50) = 5.66$ mm

Provide a 8mm plate to accommodate a 5 mm weld.

Provide a tie plate of 283 x 240 x 8 mm size and connect it with 5 mm welds as in fig (c).



Thank You